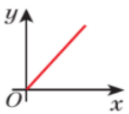
Unit 19 Higher Proportion and Graphs V345 V255 V254

When a graph of two quantities is a straight line through the origin, one quantity is directly proportional to the other.



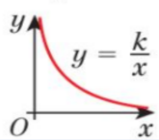
The symbol $\propto$ means 'is directly proportional to'.

If y is directly proportional to x, $y \propto x$ and $y = kx$, where k is a number, called the **constant of proportionality**.

Where k is the constant of proportionality:

- if y is proportional to the square of x then $y \propto x^2$ and $y = kx^2$
- if y is proportional to the cube of x then $y \propto x^3$ and $y = kx^3$
- if y is proportional to the square root of x then $y \propto \sqrt{x}$ and $y = k\sqrt{x}$

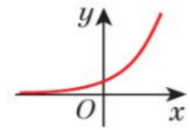
When y is **inversely proportional** to x, $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$



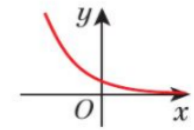
The tangent to a curved graph is a straight line that touches the graph at a point. The gradient at a point on a curve is the gradient of the tangent at that point.

Expressions of the form a^x or a^{-x}, where $a > 1$, are called **exponential functions**.

The graph of an exponential function has one of these shapes.



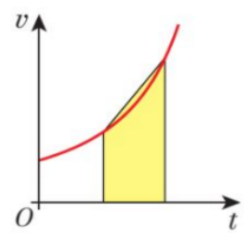
$y = a^x$ where $a > 1$ or
$y = b^{-x}$ where $0 < b < 1$
exponential growth



$y = a^{-x}$ where $a > 1$ or
$y = b^x$ where $0 < b < 1$
exponential decay

Exponential graphs intersect the y-axis at (0, 1) because $a^0 = 1$ for all values of a.

The area under a velocity–time graph shows the displacement, or distance from the starting point. To estimate the area under a part of a curved graph, draw a chord between the two points you are interested in, and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an estimate for the area under this part of the graph.



The gradient of the chord gives the average rate of change

Higher: Transformation of Graphs – Corbett Maths link: [Transformations of graphs]

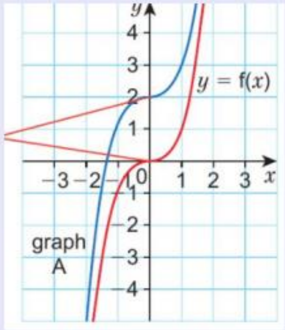
The graph of $y = \mathrm{f}(x)$ is transformed into the graph of:

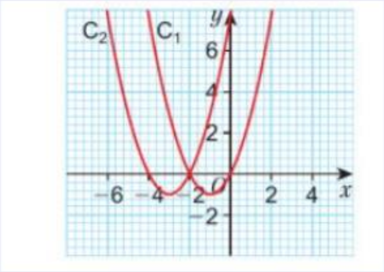
$y = \mathrm{f}(x) + a$ by a translation of a units parallel to the y-axis or a translation by $\begin{pmatrix} 0 \\ a \end{pmatrix}$

The graph of $y = \mathrm{f}(x)$ is transformed into the graph of:

$y = \mathrm{f}(x) + a$ by a translation of a units parallel to the y-axis or a translation by $\begin{pmatrix} 0 \\ a \end{pmatrix}$

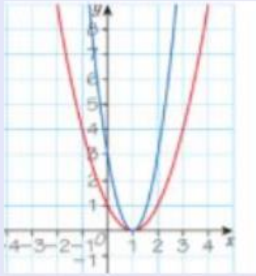
$y = \mathrm{f}(x + a)$ by a translation of $-a$ units parallel to the x-axis or a translation by $\begin{pmatrix} -a \\ 0 \end{pmatrix}$





$y = \mathrm{f}(-x)$ by a reflection in the y-axis

$y = -\mathrm{f}(x)$ by a reflection in the x-axis



$y = a\mathrm{f}(x)$ by a stretch of scale factor a parallel to the y-axis

$y = \mathrm{f}(ax)$ by a stretch of scale factor $\dfrac{1}{a}$ parallel to the x-axis