

Fraction

A mathematical expression representing the division of one integer by another.

Fractions are written as **two numbers separated by a horizontal line**.

$\frac{2}{7}$ is a 'proper' fraction

$\frac{9}{4}$ is an 'improper' or 'top-heavy' fraction

Calculating with Mixed Numbers

You need to change a **mixed number** into an **improper fraction** so you can calculate with it. **Multiply denominator** by the whole number then add **numerator** this is the new **numerator**

$$3\frac{2}{5} \text{ new numerator} = 5 \times 3 + 2 = 17$$

$$3\frac{2}{5} = \frac{17}{5}$$

Fraction of an amount

Divide by the **bottom**, **multiply** by the **top**

$$\text{Find } \frac{2}{5} \text{ of } \text{£}60 \quad 60 \div 5 = 12$$

$$12 \times 2 = 24$$

Numerator

The **top number** of a fraction

In the fraction $\frac{3}{5}$, **3 is the numerator**

Denominator

The **bottom number** of a fraction

In the fraction $\frac{3}{5}$, **5 is the denominator**

Simplifying Fractions

Divide the numerator and denominator by the highest common factor.

$$\frac{20}{45} = \frac{4}{9}$$

Equivalent Fractions

Fractions which represent the same value

$$\frac{2}{5} = \frac{4}{10} = \frac{20}{50} = \frac{60}{150} \text{ etc.}$$

Adding or Subtracting Fractions

Find the **LCM of the denominators** to find a common denominator.

Use equivalent fractions to change each fraction to the **common denominator**.

Then just **add or subtract the numerators** and keep the **denominator the same**.

$$\frac{2}{3} + \frac{4}{5}$$

Multiples of 3: 3, 6, 9, 12, **15**...

Multiples of 5: 5, 10, **15**...

LCM of 3 and 5 = 15

$$\frac{2}{3} = \frac{10}{15} \quad \frac{4}{5} = \frac{12}{15}$$

$$\frac{10}{15} + \frac{12}{15} = \frac{22}{15} = 1\frac{7}{15}$$

$$\frac{2}{3} + \frac{4}{5} = \frac{10}{15} + \frac{12}{15} = \frac{22}{15} = 1\frac{7}{15}$$

Unit Fraction

A fraction where the **numerator is one** and the denominator is positive integer.

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ etc. are examples of **unit fractions**

Mixed Number

A number formed of both an **integer part** and a **fraction part**

$3\frac{2}{5}$ is an example of a **mixed number**

Comparing Fractions

To compare fractions, they each need to be rewritten so that they have a **common denominator**.

Ascending means **smallest to biggest**

Descending means **biggest to smallest**

Put in to ascending order: $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{1}{2}$

Equivalent: $\frac{9}{12}$, $\frac{8}{12}$, $\frac{10}{12}$, $\frac{6}{12}$ Correct Order: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$

Multiplying Fractions

Multiply the numerators together and multiply the denominators together

$$\frac{3}{8} \times \frac{2}{9} = \frac{6}{72} = \frac{1}{12}$$

Dividing Fractions

'Keep it, Flip, Change it – KFC'

Keep the first fraction the same.

Flip the second fraction upside down.

Change the divide to a multiply.

$$\frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}$$

Multiply by the reciprocal of the second fraction

Try these.....

1 Work out these additions

a) $\frac{5}{7} + \frac{1}{4} =$

b) $1\frac{4}{9} + 2\frac{2}{3} =$

2 Subtract these pairs of fractions

a) $\frac{2}{5} - \frac{3}{8} =$

b) $6\frac{2}{5} - 5\frac{2}{3} =$

3 Work out

a) $\frac{7}{8} \times 24 =$

b) $\frac{4}{9} \times \frac{2}{3} =$

c) $\frac{22}{15} \times \frac{5}{11} =$

d) $\frac{1}{5} \times 2\frac{1}{2} =$

4 Serena is saving up for an Xbox.



She gets paid £21.60 a week for her paper round. She spends $\frac{1}{4}$ of this on snacks and she saves $\frac{3}{5}$ of the remainder.

How much does she have left each week to spend on other things?

5 Work out

a) $10 \div \frac{2}{5} =$

b) $\frac{5}{8} \div \frac{2}{3} =$

c) $\frac{3}{4} \div \frac{5}{16} =$

d) $2\frac{1}{3} \div \frac{4}{5} =$

6

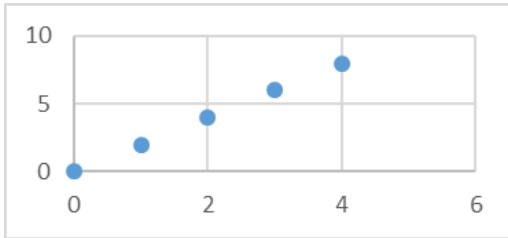


a) Round each number to the nearest integer to find an estimate of $6\frac{3}{8} - 3\frac{3}{5} \div \frac{9}{10}$

b) Use your calculator to find the exact answer

Recognising direct proportion

- Plot graphs – A straight line passing through (0,0)
- Spot simple direct proportion from a table – always (0,0) included in plot in direct proportion
- Recognising doubling tripling ... of linked x and y values



x	0	1	2	3	4
y	0	2	4	6	8

Complete calculations that show direct proportion

Recognise from a table direct proportion through quick calculation check.

$$6/4 = 12/8 = 18/12 = 1.5$$

Reasoning The table shows the amounts of CO₂ emitted from burning LPG (liquid petroleum gas).

LPG (litres)	0	6	12	18
CO ₂ (kg)	0	4	8	12

Complete calculations that **do not** show direct proportion

Recognise from a table values not in direct proportion through calculation

$$45/30 = 75/50 \quad (140/120 - \text{different values} - 1.5 \text{ and } 1.16!)$$

Problem-solving The table shows the cost of perfume in different size bottles. Are the price and volume in direct proportion?

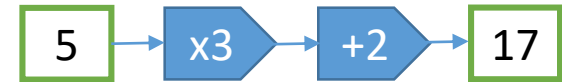
Volume (ml)	30	50	120
Price (£)	45	75	140

Solving problems with direct proportion using direct proportion graphs

Read from plots and be able to work out outside the points with calculation, e.g. 50g price would be £1000!



Plot a line given a function



- Calculate values in tables – $3x$ and add 2 for the value of y from x
- $3 \times (5) + 2 = 17$
- Plot co-ordinates and link the line with gradient (3) and cuts the y axis at (2)
- $Y = (3) x + (2)$

a) Complete the table of values below for $y = 3x + 2$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	-13	-10	-7	-4	-1	2	5	8	11	14	17

b) Plot the graph of $y = 3x + 2$

c) Use your graph to estimate the value of x when $y = 4$

Gradient definition and calculation

What is the gradient of A? $\frac{\text{change in } y}{\text{change in } x}$

Definition of gradient as the steepness of a line gradient has a-value and sign

$m = \text{change in } y / \text{change in } x$

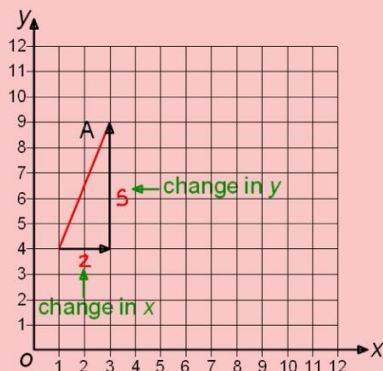
$m = 5/2$

Use symbol m for gradient

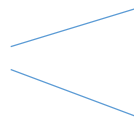
$m = 2.5$ (as 5 divided by 2)

m can be a fraction/whole or a decimal

What is the gradient of A? $\frac{\text{change in } y}{\text{change in } x} = \frac{5}{2} = 2.5$



Gradients (positive/negative)



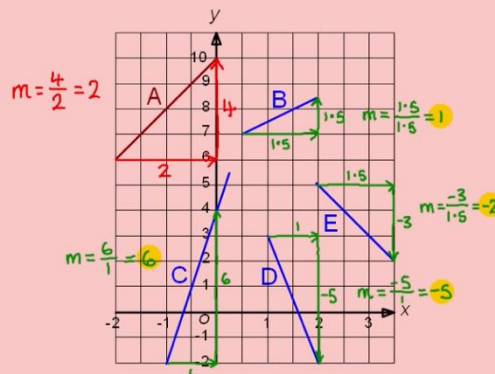
Gradients can be either positive, negative or 0 (zero)

D and E negative

Horizontal line = $m = 0$

Vertical line = undefined

Find the gradients of the lines A to E



The straight line

$$y = mx + c$$

Finding m and c on any given line

Constructing triangles on lines to find m

and

Locating c on the y axis (intercept)

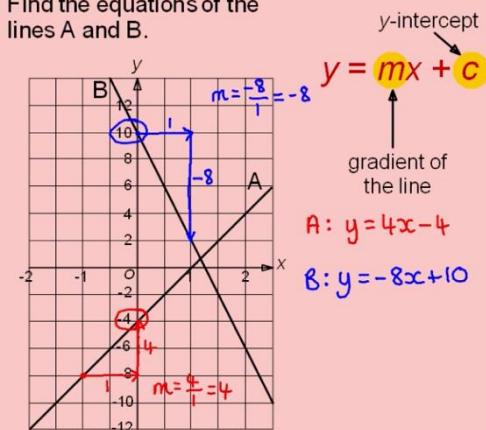
A – positive gradient

Intercept $c = 4$

B – negative gradient

Intercept $c = 10$

Find the equations of the lines A and B.



Try these.....

1

A

Minutes	Cost
1	£0.04
5	£0.20
40	£1.60

B

Minutes	Cost
2	£0.24
10	£1.00
100	£7.00

C

Minutes	Cost
10	£0.70
50	£3.50
60	£4.20

In which of the offers are the numbers in direct proportion?
In each case, explain your answer.

2

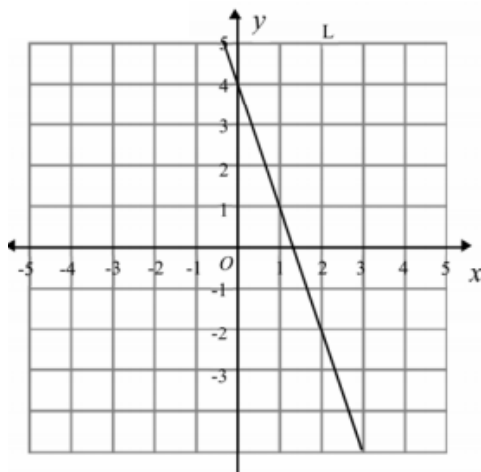
Miles	Kilometres
5	8
10	
	24
	32
50	

- Use direct proportion to complete this conversion table.
- The distance between London and Birmingham is 120 miles. Use the table to work out this distance in kilometres.
- The distance between London and Paris is 460 kilometres. Use the table to work out this distance in miles.

3

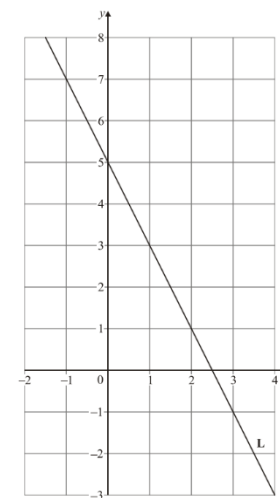
The line L is drawn on the grid below.

Find the gradient of the line.



4

Find the equation of line L





Percentage

Number of parts per 100

$$31\% \text{ means } \frac{31}{100}$$

Finding 10%

To find 10%, **divide by 10**

$$10\% \text{ of } \pounds 36 = 36 \div 10 = \pounds 3.60$$

Finding 1%

To find 1%, **divide by 100**

$$1\% \text{ of } \pounds 8 = 8 \div 100 = \pounds 0.08$$

Percentage Change

$$\frac{\text{difference}}{\text{original}} \times 100\%$$

A games console is bought for £200 and sold for £250

$$\% \text{ change} = \frac{50}{200} \times 100 = 25\%$$

Fractions to Decimals

Divide the numerator by the denominator using the bus stop method

$$\frac{3}{8} = 3 \div 8 = 0.375$$

Decimals to Fractions

Write as a fraction over 10, 100 or 1000 and simplify

$$0.36 = \frac{36}{100} = \frac{9}{25}$$

Percentages to Decimals

Divide by 100

$$8\% = 8 \div 100 = 0.08$$

Decimals to Percentages

Multiply by 100

$$0.04 = 0.4 \times 100\% = 40\%$$

Fractions to Percentages

Percentage is just a fraction out of 100. **Make the denominator 100 using equivalent fractions.** When the denominator does not go in to 100, use a calculator and **multiply the fraction by 100**

$$\frac{3}{25} = \frac{12}{100} = 12\%$$

$$\frac{9}{17} \times 100 = 52.9\%$$

Percentages to Fractions

Percentage is just a fraction out of 100. **Write the percentage over 100** and simplify

$$14\% = \frac{14}{100} = \frac{7}{50}$$



Try these.....

1 Complete the table

Fraction	Decimal	Percentage
$\frac{1}{5}$		
		37%
	0.54	

2 Express the first quantity as a percentage of the second.

a) 45cm 125cm

..... %

b) 24 minutes 3 hours 20 minutes

..... %