# YEAR 8 - ALGEBRAIC TECHNIQUES...

@whisto_maths

# Brackets, Equations & Inequalities

## What do I need to be able to do?

By the end of this unit you should be able to:

- Form Expressions
- Expand and factorise single brackets
- Form and solve equations
- Solve equations with brackets
- Represent inequalities
- Form and solve inequalities

## Keywords

**Simplify:** grouping and combining similar terms
**Substitute:** replace a variable with a numerical value
**Equivalent:** something of equal value
**Coefficient:** a number used to multiply a variable
**Product:** multiply terms
**Highest Common Factor (HCF):** the biggest factor (or number that multiplies to give a term)
**Inequality:** an inequality compares who values showing if one is greater than, less than or equal to another

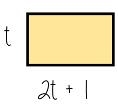
## Form expressions

For unknown variables, a letter is normally used in its place

More than – ADD
Less than/ difference – SUBTRACT

eg 4 more than t → t + 4
8 less than k → k - 8

Only similar terms can be grouped together




eg Find the perimeter of this shape
(Perimeter = length around outside of shape)

t + 2t + 1 + t + 2t + 1 → 6t + 2

## Directed numbers

\+ \+ → \+

\- \- → \+

\+ \- → \-

\- \+ → \-

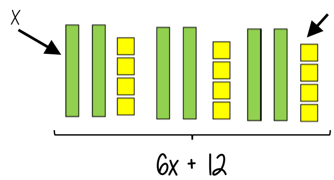
eg $a = -5$ and $b = 2$
$a^2 = a \times a = -5 \times -5 = 25$
$b + a = 2 + -5 = -3$

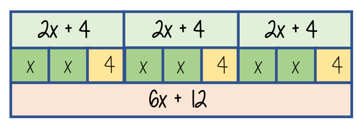
## Multiply single brackets

$3(2x + 4)$

| | 2x | 4 |
|---|---|---|
| 3 | 3 x 2x<br>6x | 3 x 4<br>12 |

6x + 12

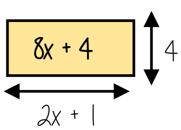



Different representations of 3(2x+4) = 6x + 12

## Factorise into a single bracket

8x + 4




← Try and make this the **highest** common factor

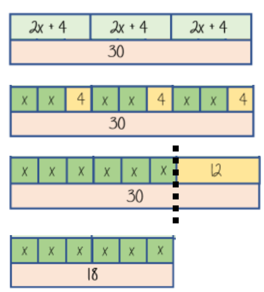
The two values **multiply** together (also the area) of the rectangle

$8x + 4 \equiv 4(2x + 1)$

Note:
$8x + 4 \equiv 2(4x + 2)$
This is factorised but the HCF has not been used

## Solve equations with brackets

$3(2x + 4) = 30$




3 (2x + 4) = 30

Expand the brackets

6x + 12 = 30
-12 -12

Substitute to check your answer.
This could be negative or a fraction or decimal

6x = 18
÷ 6 ÷ 6


x = 3

## Simple Inequalities

< less than
≤ Less than or equal to
> More than
≥ More than or equal to

x < 10
Say this out loud
"x is a value less than 10"

10 > x
Say this out loud
"10 is more than the value"

Note:
x<10 and 10>x represent the same values

$x + 2 \leq 20$
"my value + 2 is less than or equal to 20"
$x \leq 18$
The biggest the value can be is 18

## Form and solve inequalities

Two more than treble my number is greater than 11

Find the possible range of values

**Form**

x → x3 → +2 → 11

$3x + 2 > 11$

**Solve**

x ← ÷3 ← -2 ← 11

$x > 3$

**Check**

This would suggest any value bigger than 3 satisfies the statement
3 x 3 + 2 = 11 ✓   10 x 3 + 2 = 32 ✓

## Algebraic constructs

**Expression**
A sentence with a minimum of two numbers and one maths operation

**Equation**
A statement that two things are equal

**Term**
A single number or variable

**Identity**
An equation where both sides have variables that cause the same answer includes ≡

**Formula**
A rule written with all mathematical symbols
eg area of a rectangle A = b x h

# YEAR 8 - ALGEBRAIC TECHNIQUES...

# Sequences

@whisto_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Generate a sequence from term to term or position to term rules
- Recognise arithmetic sequences and find the nth term
- Recognise geometric sequences and other sequences that arise

## Keywords

**Sequence**: items or numbers put in a pre-decided order
**Term**: a single number or variable
**Position**: the place something is located
**Linear**: the difference between terms increases or decreases (+ or -) by a constant value each time
**Non-linear**: the difference between terms increases or decreases in different amounts, or by x or ÷
**Difference**: the gap between two terms
**Arithmetic**: a sequence where the difference between the terms is constant
**Geometric**: a sequence where each term is found by multiplying the previous one by a fixed non zero number

## Linear and Non Linear Sequences

**Linear Sequences** – increase by addition or subtraction and the same amount each time

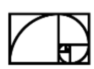
**Non-linear Sequences** – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

Fibonacci Sequence – look out for this type of sequence

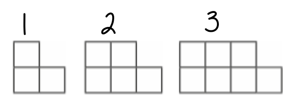
0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms.



## Sequence in a table and graphically

**Position**: the place in the sequence




"The term in position 3 has 7 squares"

3 5 7

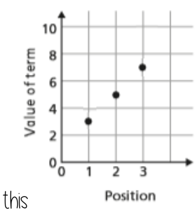
**Term**: the number or variable
(the number of squares in each image)

In a table

| Position | 1 | 2 | 3 |
|---|---|---|---|
| Term | 3 | 5 | 7 |

+2 +2

Graphically




Because the terms increase by the same addition each time this is **linear** – as seen in the graph

## Sequences from algebraic rules

This is substitution!

$3n + 7$

This will be linear - note the single power of n. The values increase at a constant rate

$3n^2 + 7$

This is not linear as there is a power for n

$2n - 5$ →

Substitute the number of the term you are looking for in place of 'n'

eg
1st term = $2(1) - 5 = -3$
2nd term = $2(2) - 5 = -1$
100th term = $2(100) - 5 = 195$

## Checking for a term in a sequence

Form an equation

Is 201 in the sequence $3n - 4$?

$3n - 4 = 201$

Algebraic rule

Term to check

Solving this will find the position of the term in the sequence.
ONLY an integer solution can be in the sequence.

## Complex algebraic rules

Misconceptions and comparisons

$2n^2$

2 times whatever n squared is

eg
1st term = $2 \times 1^2 = 2$
2st term = $2 \times 2^2 = 8$
100th term = $2 \times 100^2 = 2000$

$(2n)^2$

2 times n then square the answer

eg
1st term = $(2 \times 1)^2 = 4$
2st term = $(2 \times 2)^2 = 16$
100th term = $(2 \times 100)^2 = 40000$

$n(n + 5)$ ←

e.g
1st term = $1(1 + 5) = 6$
2st term = $2(2 + 5) = 14$
100th term = $100(100 + 5) = 10500$

You don't need to expand the expression

## Finding the algebraic rule

This is the 4 times table → 4, 8, 12, 16, 20...

$4n$

7, 11, 15, 19, 22 ←

This has the same constant difference – but is 3 more than the original sequence

$4n + 3$

$4n + 3$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

# YEAR 8 - ALGEBRAIC TECHNIQUES...

@whisto_maths

# Indices

## What do I need to be able to do?

By the end of this unit you should be able to:

- Add/ Subtract expressions with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices

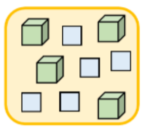
## Keywords

**Base**: The number that gets multiplied by a power
**Power**: The exponent – or the number that tells you how many times to use the number in multiplication
**Exponent**: The power – or the number that tells you how many times to use the number in multiplication
**Indices**: The power or the exponent.
**Coefficient**: The number used to multiply a variable
**Simplify**: To reduce a power to its lowest term
**Product**: Multiply

## Addition/ Subtraction with indices

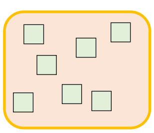
Coefficient → $\underbrace{5x^2}_{\text{Term}} + \underbrace{4x^4}_{\text{Term}}$ ← Power

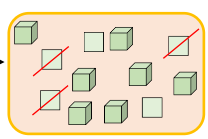
Expression



Each square represents $x^2$ and each cube represents $x^4$

Only similar terms can be simplified
If they have different powers, they are unlike terms

$5x^2 + 2x^2 \rightarrow$  $\rightarrow 7x^2$

$5x^2 + 6x^4 - 3x^2 + x^4 \rightarrow$  $\rightarrow 2x^2 + 7x^4$

## Multiply expressions with indices

$4b \times 3a$
$\equiv 4 \times b \times 3 \times a$
$\equiv 4 \times 3 \times b \times a$
$\equiv 12ab$

$5t \times 9t$
$\equiv 5 \times t \times 9 \times t$
$\equiv 5 \times 9 \times t \times t$
$\equiv 45t^2$

$2b^4 \times 3b^2$
$\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b$
$\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b$
$\equiv 6b^6$

There are often misconceptions with this calculation but break down the powers

## Divide expressions with indices

$$\frac{24}{36} \rightarrow \frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 2 \times \cancel{3}} \rightarrow \frac{2}{3}$$

$$\frac{5a^3b^2}{15ab^6} \rightarrow \frac{\cancel{5} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b}}{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times b \times b \times b \times b} \rightarrow \frac{a^2}{3b^4}$$

Cross cancelling factors shows cancels the expression

$$\frac{23a^7y^2}{5db^6}$$

This expression cannot be divided (cancelled down) because there are no common factors or similar terms

## Addition/ Subtraction laws for indices

$3^5 \times 3^2$
$= \underbrace{(3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)}$ $\rightarrow 3^7$

The base number is all the same so the terms can be simplified

**Addition law for indices**

$$a^m \times a^n = a^{m+n}$$

$3^5 \div 3^2 \rightarrow 3^3$

$$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \rightarrow \frac{3^3}{3^0} \rightarrow \frac{3^3}{1}$$

**Subtraction law for indices**

$$a^m \div a^n = a^{m-n}$$

# YEAR 8 - DEVELOPING NUMBER...
# Fractions & Percentages

@whisto_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Convert between FDP less than and more than 100.
- Increase or decrease using multipliers.
- Express an amount as a percentage.
- Find percentage change.

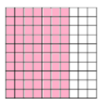
## Keywords

**Percent**: parts per 100 – written using the % symbol
**Decimal**: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.
**Fraction**: a fraction represents how many parts of a whole value you have.
**Equivalent**: of equal value.
**Reduce**: to make smaller in value.
**Growth**: to increase/ to grow.
**Integer**: whole number, can be positive, negative or zero.
**Invest**: use money with the goal of it increasing in value over time (usually in a bank).

## Convert FDP

R

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares, 70 "hundredths" = 7 "tenths", 0.7 →



→ 70 hundredths = 70%

Using a calculator

S⇌D

This will give you the answer in the simplest form

Convert to a decimal

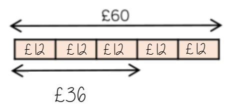
× 100 converts to a percentage

Be careful of recurring decimals
eg $\frac{1}{3}$ = 0.3333333
= $0.\dot{3}$
The dot above the 3

## Fraction/ Percentage of amount

R

Find $\frac{3}{5}$ of £60



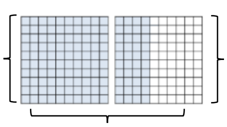

Remember
$\frac{3}{5}$ = 60%

10% of £60 = £6
50% of £60 = £30
60% of £60 = £36

Remember
$\frac{3}{5}$ = 60% = 0.6

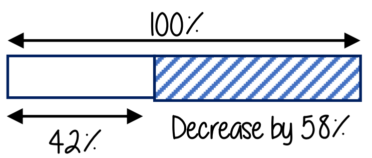
60% of £60
= 0.6 x 60
= £36

## Convert FDP < and > 100%




100% + 40%
1 + 0.40
= 1.40

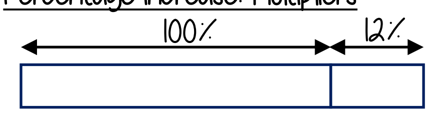
## Percentage decrease: Multipliers




100% - 58% = 42%
1.00 – 0.58 = 0.42

Multiplier
Less than 1

## Percentage increase: Multipliers

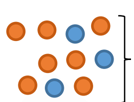



100% + 12% = 112%
1.00 + 0.12 = 1.12

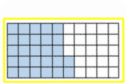
Multiplier
More than 1

## Express as a % - Non-calculator

Percent – per hundred



7 per every 10 are orange: $\frac{7}{10}$ → This means that 70 per every 100 are orange: $\frac{70}{100}$ → 70%



27 per every 50 shaded: $\frac{27}{50}$ → 54 per every 100 shaded: $\frac{54}{100}$ → 54%

Denominator 100

Equivalent fractions

## Express as a % - Calculator

Rosie

$\frac{13}{30}$ → $\frac{\blacksquare}{\square}$ → $\frac{13}{30}$ → × 100 → 43.3333...%
43%

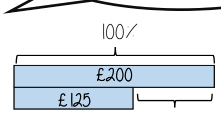
This the same as 13 ÷ 30

Can't use equivalence easily to find 'per hundred'

Decimal percentages are still a percentage

## Percentage change

I bought a phone for £200.
A year later sold it for £125



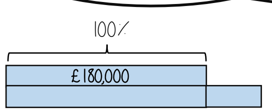

All values of change compare to the ORIGINAL value

Percentage loss

$\frac{75}{200}$ × 100 = 37.5%

$\frac{\text{Difference in value}}{\text{Original value}}$ × 100

I bought a house for £180,000, I later sold it for £216,000




Percentage profit

Money made (profit value) $\frac{36000}{180000}$ × 100 = 20%

## Choose appropriate method

The language and wording of the question is the key

Have you represented the question in a bar model?
Can you use a calculator?

# YEAR 8 - DEVELOPING NUMBER...

# Standard Form

@whisto_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Write numbers in standard form and as ordinary numbers
- Order numbers in standard form
- Add/ Subtract with standard from
- Multiply/ Divide with standard form
- Use a calculator with standard form

## Keywords

**Standard (index) Form**: A system of writing very big or very small numbers
**Commutative**: an operation is commutative if changing the order does not change the result.
**Base**: The number that gets multiplied by a power
**Power**: The exponent – or the number that tells you how many times to use the number in multiplication
**Exponent**: The power – or the number that tells you how many times to use the number in multiplication
**Indices**: The power or the exponent.
**Negative**: A value below zero.

## Positive powers of 10

1 billion – 1 000 000 000
$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

Addition rule for indices $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices $10^a \div 10^b = 10^{a-b}$

## Standard form with numbers > 1

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

**Example**

$3.2 \times 10^4$
$= 3.2 \times 10 \times 10 \times 10 \times 10$
$= 32000$

**Non-example**

$0.8 \times 10^4$

$5.3 \times 10^{0.7}$

## Negative powers of 10

0.001

$1 \times \frac{1}{1000}$

$1 \times 10^{-3}$

| 10 | 1 | • | $\frac{1}{10}$ | $\frac{1}{100}$ | $\frac{1}{1000}$ |
|---|---|---|---|---|---|
| $10^1$ | $10^0$ | • | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ |
| 0 | 0 | • | 0 | 0 | 1 |

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

## Numbers between 0 and 1

0.054
$= 5.4 \times 10^{-2}$

| 1 | • | $\frac{1}{10}$ | $\frac{1}{100}$ | $\frac{1}{1000}$ |
|---|---|---|---|---|
| $10^0$ | • | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ |
| 0 | • | 0 | 5 | 4 |

A negative power does not mean a negative answer – it means a number closer to 0

## Order numbers in standard form

| $10^2$ | $10^1$ | $10^0$ | • | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ | $10^{-4}$ |
|---|---|---|---|---|---|---|---|

| $6.4 \times 10^{-2}$ | $2.4 \times 10^2$ | $3.3 \times 10^0$ | $1.3 \times 10^{-1}$ |
|---|---|---|---|
| 0.064 | 240 | 1 | 0.13 |

Look at the power first will the number be = > or < than 1

Use a place value grid to compare the numbers for ordering

## Mental calculations

$6.4 \times 10^2 \times 1000$ Not in Standard Form
$= 6.4 \times 10^2 \times 10^3$ Use addition for indices rule
$= \underline{6.4 \times 10^5}$

$8 \times 10^5 \times 3$
$= 24 \times 10^5$ Not in Standard Form
$= 2.4 \times 10^1 \times 10^5$ Use addition for indices rule
$= \underline{2.4 \times 10^6}$

$(2 \times 10^3) \div 4$ Divide the values
$= (2 \div 4) \times 10^3$
$= \underline{0.5 \times 10^3}$

Remember the layout for standard form

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

## Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$6 \times 10^5 + 8 \times 10^5$

Method 1

$= 600000 + 800000$
$= 1400000$
$= \underline{1.4 \times 10^5}$

More robust method
Less room for misconceptions
Easier to do calculations with negative indices
Can use for different powers

Method 2

$= (6 + 8) \times 10^5$
$= 14 \times 10^5$ ← This is not the final answer
$= 1.4 \times 10^1 \times 10^5$
$= \underline{1.4 \times 10^5}$

Only works if the powers are the same

## Multiplication and division

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$ Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \times 10^5 \div 10^3$

$= \underline{5 \times 10^2}$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

Revisit addition and subtraction laws for indices – they are needed for the calculations

Addition law for indices
$a^m \times a^n = a^{m+n}$

Subtraction law for indices
$a^m \div a^n = a^{m-n}$

## Using a calculator

$14 \times 10^5 \times 3.9 \times 10^3$

Use a calculator to work out this question to a suitable degree of accuracy

Input 14 and press x10^x Then press 5 (for the power)
Press ×
Input 3.9 and press x10^x Then press 3 (for the power)
Press =

This gives you the solution

Click calculator for video tutorial

To put into standard form and a suitable degree of accuracy

Press SHIFT SETUP and then press 7 for sci mode

Choose a degree of accuracy so in most cases press 2

Answer: $5.5 \times 10^8$

# YEAR 8 - DEVELOPING NUMBER...

# Number Sense

@whisto_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Round numbers to powers of 10 and 1 sf
- Round numbers to any dp
- Estimate solutions
- Calculate using order of operations
- Calculate with money, units of measurement and time

## Keywords

**Significant:** Place value of importance
**Round:** Making a number simpler but keeping its value close to what it was.
**Decimal:** Place holders after the decimal point.
**Overestimate:** Rounding up – gives a solution higher than the actual value
**Underestimate:** Rounding down – gives a solution lower than the actual value.
**Metric:** A system of measurement.
**Balance:** The amount of money in a bank account
**Deposit:** Putting money into a bank account.

## Round to powers of 10 and 1 sig figure

R

If the number is halfway between we "**round up**"

5495 to the nearest 1000

5000 ↑ 6000

5475 to the nearest 100

5400 ↑ 5500

5475 to the nearest 10

5470 ↑ 5480

370 to 1 significant figure is 400
37 to 1 significant figure is 40
3.7 to 1 significant figure is 4
0.37 to 1 significant figure is 0.4
0.00037 to 1 significant figure is 0.0004

Round to the first non-zero number

## Round to decimal places

2.46192 ← Focus on the numbers **after** the decimal point

"To 1dp" – to one number after the decimal
"To 2dp" – to two numbers after the decimal

2.46192 (to 1dp) - Is this closer to 2.4 or 2.5

2.4 ↑ 2.5

2.4 | 6192 This shows the number is closer to 2.5

2.46192 (to 12dp) - Is this closer to 2.46 or 2.47

2.46 ↑ 2.47

2.46 | 192 This shows the number is closer to 2.46

## Estimate the calculation

Round to 1 significant figure to estimate

$4.2 + 6.7 \approx 4 + 7 \approx 11$ This is an **overestimate** because the 6.7 was rounded up more

The equal sign changes to show it is an estimation

$21.4 \times 3.1 \approx 20 \times 3 \approx 60$ This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths – it helps you identify calculation errors.

## Order of operations

R

**Brackets** Operations in brackets are calculated first

**Other** operations e.g. powers, roots,

**Multiplication/ Division**

They are carried out in the order from left to right in the question

**Addition/ Subtraction**

They are carried out in the order from left to right in the question

## Calculations with money

**Debit** - You have £0 or more in an account

**Credit** - You have less than £0 in an account

Money calculations are to 2dp

Using a calculator – ensure you are working in the correct units.
£1.30 + 50p = 130 + 50 (in pence)
= 1.30 + 0.50 (in pounds)

£1 = 100p

## Units are important:

Useful Conversions

mm $\xrightarrow{\times 10}$ cm $\xrightarrow{\times 100}$ m $\xrightarrow{\times 1000}$ km

km $\xrightarrow{\div 1000}$ m $\xrightarrow{\div 100}$ cm $\xrightarrow{\div 10}$ mm

g $\xrightarrow{\times 1000}$ kg $\xrightarrow{\times 1000}$ T

T $\xrightarrow{\div 1000}$ kg $\xrightarrow{\div 1000}$ g

ml $\xrightarrow{\times 1000}$ L

L $\xrightarrow{\div 1000}$ ml

## Metric measures of length

Kilo = 1000 x meter

Centi - $\frac{1}{100}$ x meter

Milli - $\frac{1}{1000}$ x meter

## Units of weight/ capacity

Weight = g, kg, t
Capacity (volume of liquid) = ml, L

## Time and the calendar

1 Year – the amount of time it takes Earth to go around the sun 365 (and a quarter) **days**

Leap Year – **366 days** (every 4 years)

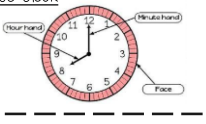
12 Months = one year = 52 weeks
31 days – **Jan, March, May, July, Aug, Oct, Dec**
30 days – **April, June, Sept, Nov**
28 days – **Feb** (29 leap year)

1 week – 7 days
Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday

1 day – 24 hours
1 hour - 60 minutes
1 minute – 60 seconds

Use a number line for time calculations!

Analogue Clock




12-hour clock

- Use am (morning) and pm (afternoon)
- Only use hour times up to 12

Digital Clock (24-hour times)


24-hour clock

- 0-11 (morning hours)
- 12-23 (afternoon hours)