Exmouth Community College
Academy Trust

Year 10 – Maths / Unit 1 (H) – Number - Autumn Term

HCF and LCM

V219 V218

(Highest Common Factor and Lowest Common Multiple)

HCF - this is largest number that divides exactly into 2 or more numbers. E.g. HCF or 12 and 20 = 4
LCM - this is the smallest number that is in the times table of 2 or more numbers. E.g. LCM of 12 and 20 = 60

Product of Prime Factors

V219

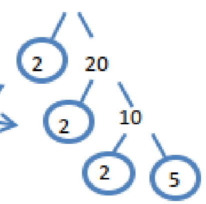
This is finding all the prime numbers that would multiply to give our number. It is often shown using a factor tree ('tree thingy').

V223

Eg. 40 as a product of prime factors

We are looking for the prime numbers that go into our number

40 = 2 x 2 x 2 x 5



We can then multiply our prime numbers together to get our original number

Using product of prime factors to find our HCF and LCM

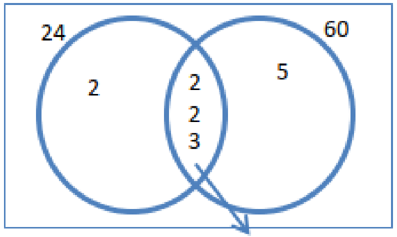
Example: Find the HCF and LCM of 24 and 60

Step 1:
24 = 2 x 2 x 2 x 2
60 = 2 x 2 x 3 x 5

Write each number as a product of prime factors

Step 2: Draw a Venn Diagram

V224



Place you prime factors into your Venn diagram

The HCF of 24 and 60 = 2 x 2 x 3 = 12

Multiply the common prime factors

The LCM of 24 and 60 = 2 x 2 x 2 x 3 x 5 = 120

Multiply all the prime factors

Laws of Indices

V17

Index/power

3^3

base

When multiplying indices with the same base, you ADD the indices

Multiplying Indices

$y^3 \times y^3 = y \times y \times y \times y \times y \times y = y^6$

Dividing Indices

$y^6 \div y^4 = \frac{y \times y \times y \times y \times y \times y}{y \times y \times y \times y} = y^2$

When dividing indices with the same base, you SUBTRACT the indices

Power to another power (brackets)

$(y^3)^2 = (y \times y \times y)^2$
$= y \times y \times y \times y \times y \times y = y^6$

With brackets just MULTIPLY your indices

Zero Indices

$y^0 = 1$

Anything to the power of 0 always equals 1

Negative Indices

V175

$y^{-1} = \frac{1}{y}$

$y^{-2} = \frac{1}{y^2}$

The negative sign means 'one over' the base number

e.g. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Fractional Indices

$y^{\frac{2}{3}} = (\sqrt[3]{y})^2$

$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 4$

The denominator of the fractional power becomes a root and the numerator becomes a power

V173

Standard Form

V300 V301 V302 V303

A number is in standard form when it is in the form A x 10^n , $where\ 1 \leq A < 10$.

For example, 63000 = 6.3 x 10^4. This is in standard form because 6.3 is between 1 and 10. 63 x10^4 is not in standard form as 63 is not between 1 and 10.

Examples
45 000 000 000 = 4.5 x 10^{10}
0.0000000000091 = 9.1 x 10^{-12}

Standard form is used so very large or very small numbers can be written out easily.

Surds

A surd is a number written exactly using square or cube roots.
For example $\sqrt{3}$ and $\sqrt{5}$ are surds. $\sqrt{4}$ and $\sqrt[3]{27}$ are not surds, because $\sqrt{4}$ = 2 and $\sqrt[3]{27}$ = 3.

Multiplying Surds

$\sqrt{m} \times \sqrt{n} = \sqrt{m \times n} = \sqrt{mn}$

E.g. $\sqrt{3} \times \sqrt{2} = \sqrt{3 \times 2} = \sqrt{6}$

Dividing Surds

$\sqrt{m} \div \sqrt{n} = \sqrt{\frac{m}{n}}$

E.g. $\sqrt{12} \div \sqrt{3} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$

V305 V306 V307 V308

Corbett Maths video links:★ V7 ★V13★ V288

n^{th} term:

Example: For the following sequence, the first term (n = 1) is 2.
The 2nd term (n = 2) is 5.

Positions (n numbers) →	1	2	3	4	5	6	. . . . n
TERMS →	2	5	8	11	14	17	. . . . .

+3 +3 +3 +3 +3

So we try rule: $n^{th\ term} = 3n$. Testing the rule with n = 1 (1st term) gives 3,
and we know 1st term should be 2, so we need an extra correction to rule of -1

So rule is: $\boxed{t_n = 3n - 1}$ 67th term is $t_{67} = 3 \times 67 - 1 = 200$

Simplifying expressions:

Gather together like terms,
eg. **3e** + *2* + **4e** – *8* = **7e** + *6*

Substitution:

Just like in sport, *substitution* means *swapping* one thing for another – but instead of a fresh player for a tired player, it's swapping a number for a letter.

If ⚽ = 5

then: ⚽ + 4 = 5 + 4 = 9

6 x ⚽ = 6 x 5 = 30

⚽ / 5 = 5 ÷ 5 = 1

Rather than drawing a football every time, they'd just use the letter "f"

When the expressions or formulae become a bit more complicated, it's *essential* that you follow the rules of BODMAS/BIDMAS:

e.g. If g = 10: 5 + 3g = 5 + 3 x 10
= 5 + 30
= 35

Classic exam question:

Bob works shifts in a café, where he get £6 a hour,
plus a £5 travel bonus each day.

(a) Write a formula to describe his pay P for a day's shift of h hours: P = 6h + 5
(b) Use this formula to find his pay for a 7 hour shift: P = 6h + 5 = 6 x 7 + 5 = 42 + 5 = £47

Solving equations:

BALANCE METHOD:

You can use this on any equation, whether the unknown is on one side, or both

You can do whatever to like, so long as you do the *same* to both sides:

4f + 3 = 2f + 23

4f + 3	**=**	**2f + 23**	**[take 2f from each side]**
2f + 3	**=**	**23**	**[take 3 from each side]**
2f	**=**	**20**	**[divide both sides by 2]**
f	**=**	**10**	

If you want to get rid of something negative, ADD that same amount to both sides

Factorising

expanding brackets

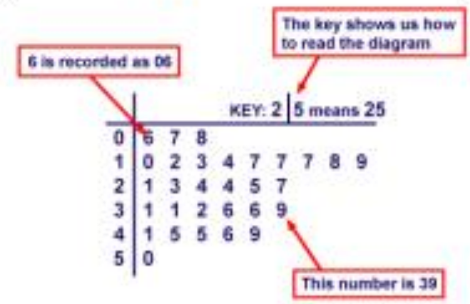
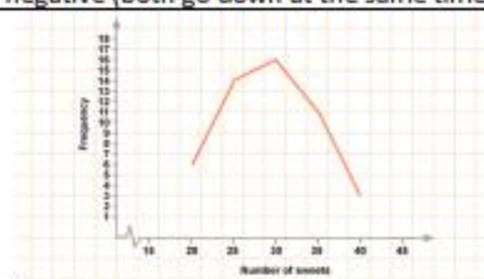
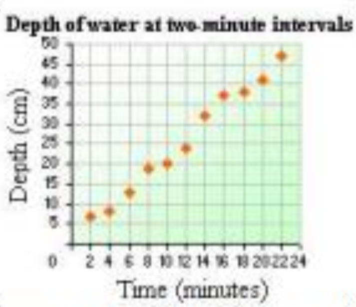
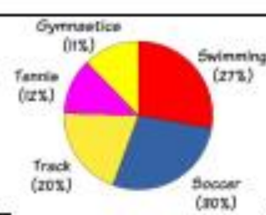
3 (2t + 5) **6t + 15**

factorising

Expanding (2a+3)(4a+2)

	2a	+3
4a	$8a^2$	+12a
+2	+4a	+6

$8a^2 + 16a + 6$

Mean: add up the numbers and divide by how many there are	*Median*: the 'middle' number. Order the numbers from smallest to largest and it's in the middle
Mode: the most commonly occurring number	*Range:* the difference between the largest and smallest numbers.
Stem & Leaf Diagram: a pictorial representation of grouped data The stem and leaf diagram is formed by splitting the numbers into two parts - in this case, tens (stem) and units (leaves). This information is given to us in the Key. It is usual for the numbers to be ordered.	
Frequency: the number of data points that fit into a category	*Correlation:* a mutual relationship or connection between two or more things. Can be positive (both go up at the same time) or negative (both go down at the same time).
Frequency polygon: a line graph that plots the frequency against the mid point of the group	
Modal Class: the class/group that has the highest frequency	*Medians in frequency tables:* if the total frequency is n then the median point lies in the class containing the $\frac{n+1}{2}$
	*Scatter graph:* used to represent and compare two sets of data. By looking at a scatter diagram, we can see whether there is any connection (correlation) between the two sets of data.
Line of best fit: A line of best fit is a straight line drawn through the center of a group of data points plotted on a scatter plot. Scatter plots depict the results of gathering data on two variables.	*Outlier:* a point which does not fit the overall pattern of a scatter graph.
Pie chart: a type of graph in which a circle is divided into sectors that each represent a proportion of the whole	

Corbett Maths video links: V271 V239 V234

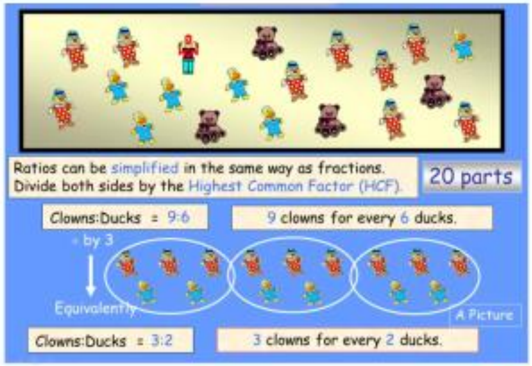
Fractions: Ratio, simplifying:

Reciprocal of n is $\frac{1}{n}$

To add and subtract mixed numbers, usually easier to convert them into *improper* (top-heavy) fractions, e.g.:

$$2\frac{1}{3}+5\frac{1}{4}=\frac{7}{3}+\frac{21}{4}$$

(then use Battenburg method)



Battenburg: adding

1. Draw the battenburg grid.
2. Put the fractions on the side, (left to right, top to bottom).
3. Eat the top left corner (cross it out).
4. Do the multiplications.
5. "**ADD** the peanut" (the yellow ones below).
6. Peanut answer is numerator, the remaining number is denominator.
7. Simplify the fraction, if possible.

$$\frac{1}{4}+\frac{1}{3}=\frac{7}{12}$$

	1	4
1	X	4
3	3	12

Divide top and bottom of fraction with the HCF that they share

Battenburg: subtracting

1. Draw the battenburg grid.
2. Put the fractions on the side, (left to right, top to bottom).
3. Eat the top left corner (cross it out).
4. Do the multiplications.
5. "**SUBTRACT** the peanut" (the yellow ones below).
6. Peanut answer is numerator, the remaining number is denominator.
7. Simplify the fraction, if possible.

$$\frac{1}{4}-\frac{1}{3}=\frac{1}{12}$$

	1	4
1	X	4
3	3	12

Divide top and bottom of fraction with the HCF that they share

Percentages of amounts

<u>Calculator allowed</u>?

Turn % into decimal (÷100) and "of" means "multiply".

e.g. 30% of £54 = 30 ÷100 x 54 = £16.20

e.g. 28% of £40 = 28 ÷100 x 40 = £11.20

<u>Reverse percentages</u>:

Use the logic of function machines, which can be run backwards.
You need to figure out the forwards multiplier first.

e.g. $30 dress reduced by 20%:

$30 → X 0.8 → $24

e.g. Sale price after 30% discount = £28

? → X 0.7 → £28

Original price £40 ← ÷ 0.8 ← £28

<u>Calculator *not* allowed?</u>

10% is your starting point.
10% means "a tenth of the amount"
(because 10% = 10/100 = 1/10)

You can work out all the other percentages you need by scaling up or down from 10%

e.g. 30% of £54?

10% = £5.40 (a tenth of 54 = 54/10)
20% = £10.80 (20% is double 10%)
30% = £16.20 (30% = 10% + 20%)

e.g. 28% of £40?

10% = £4
1% = 40p (divide 10% by 10)
2% = 80p (double 1%)
5% = £2 (half 10%)
20% = £8 (double 10%)

28% = these 4 added together, = £11.20

Fractions, decimals, percentages conversion

numerator ÷ denominator **X 100**

Example:

Fraction **24/100 = 6/25** → **decimal** **0.24** → **percentage** **24%**

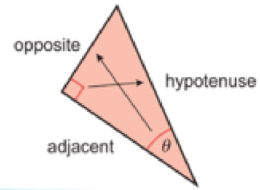
Write as fraction out of 10, 100, or 1000, and simplify **÷ 100**

<u>Some examples</u>:

1/10 =	10/100 =	0.1 =	10%
1/5 =	20/100 =	0.2 =	20%
3/10 =	30/100 =	0.3 =	30%
9/20 =	45/100 =	0.45 =	45%

People often assume a % cannot be over 100, but it can (just like a fraction can be improper* and a decimal can be over 1)

* top-heavy



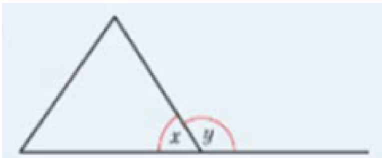
In a right-angled triangle, the longest side is called the hypotenuse and is opposite the right-angle.

The side opposite the angle θ is called the **opposite.**

The side that is next to angle θ is the **adjacent**.

When one side of a triangle is extended at the vertex, it forms an **exterior** angle.
x is the **interior** angle.
y is the **exterior** angle.

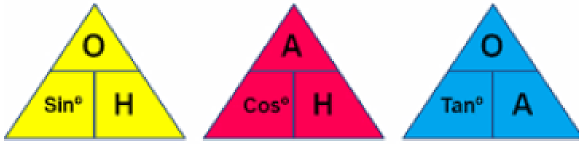
$$x + y = 180^o$$



The sum of the interior angles of a polygon with n sides = (n-2) x 180^0

The sum of the **exterior** angles of a polygon is always 360^0

SOH CAH TOA

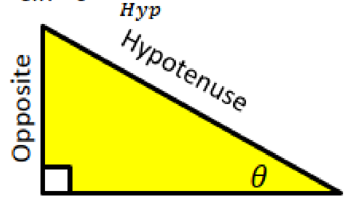


Sine Ratio

$Opp = sin\theta$ x Hyp

$Hyp = \frac{Opp}{sin\,\theta}$

$sin^{-1}\,\theta = \frac{Opp}{Hyp}$

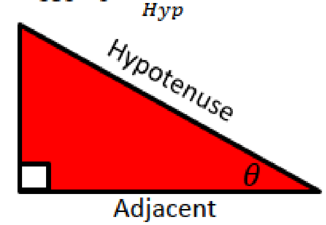


Cosine Ratio

$Adj = cos\theta$ x Hyp

$Hyp = \frac{Adj}{cos\,\theta}$

$cos^{-1}\,\theta = \frac{Adj}{Hyp}$

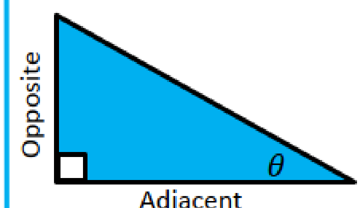


Tangent Ratio

$Opp = tan\theta$ x Adj

$Adj = \frac{Opp}{tan\,\theta}$

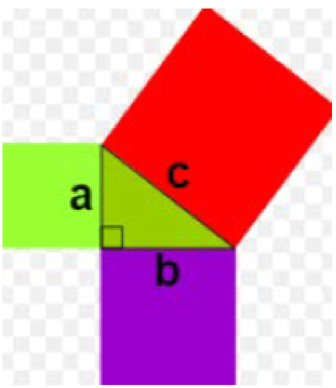
$tan^{-1}\,\theta = \frac{Opp}{Adj}$



To get sin^{-1}, cos^{-1} and tan^{-1} press shift on the calculator and then the corresponding ratio.

Pythagoras' Theorem

$a^2 + b^2 = c^2$



To find **hypotenuse**:
Square side a
Square side b
Add together
Square root

To find shorter side:
Square side c
Square side a or b
Subtract a or b from c
Square root

★ V257

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	

The exact **sine**, **cosine** and **tangent** of some angles are in this table.

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