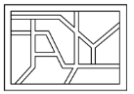


# YEAR 8 - PROPORTIONAL REASONING...

## Ratio and Scale

@whisto\_maths



### What do I need to be able to do?

By the end of this unit you should be able to:

- Simplify any given ratio
- Share an amount in a given ratio
- Solve ratio problems given a part

Solutions should be modelled, explained and solved

### Keywords

**Ratio:** a statement of how two numbers compare

**Equal Parts:** all parts in the same proportion, or a whole shared equally

**Proportion:** a statement that links two ratios

**Order:** to place a number in a determined sequence

**Part:** a section of a whole

**Equivalent:** of equal value

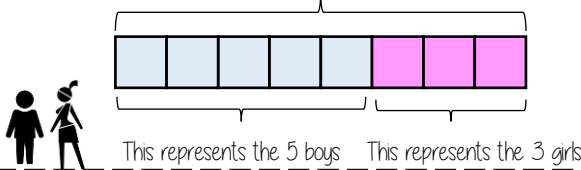
**Factors:** integers that multiply together to get the original value

**Scale:** the comparison of something drawn to its actual size.

### Representing a ratio

"For every 5 boys there are 3 girls"

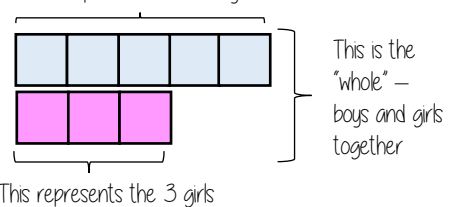
This is the "whole" - boys and girls together



5:3

This represents the 5 boys

Double Number Line



### Order is Important

"For every dog there are 2 cats"



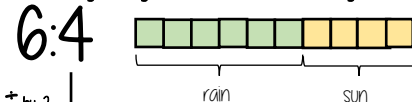
The ratio has to be written in the same order as the information is given

e.g. 2:1 would represent 2 dogs for every 1 cat ✗

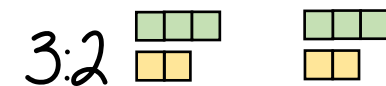
### Simplifying a ratio

Cancel down the ratio to its lowest form

"For every 6 days of rain there are 4 days of sun"



+ by 2 ↓



3:2

"For every 3 days of rain there are 2 days of sun" - when this happens twice the ratio becomes 6:4.

Find the biggest common factor that goes into all parts of the ratio

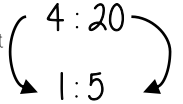
For 6 and 4 the biggest factor (number that multiplies into them is 2)

### Ratio In (or n:1)

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that this part has to be 1 unit. Therefore Divide by 4



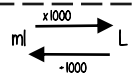
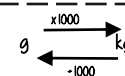
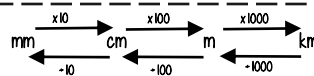
This side has to be divided by 4 too - to keep in proportion

\*\*The n part does not have to be an integer for this type of question

### Units are important:

When using a ratio - all parts should be in the same units

Useful Conversions



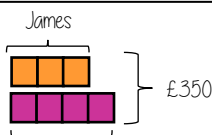
### Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4. Work out how much each person earns

Model the Question

James: Lucy

3:4



Lucy

£350 ÷ 7 = £50

□ = one part = £50

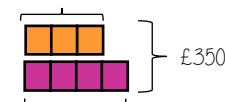
Find the value of one part

Whole: £350  
7 parts to share between (3 James, 4 Lucy)

Put back into the question

James: Lucy

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

(x 50) 3:4 (x 50)  
£150:£200

### Finding a value given 1:n (or n:1)

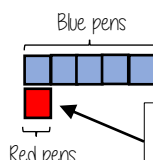
Inside a box are blue and red pens in the ratio 5:1. If there are 10 red pens how many blue pens are there?

Model the Question

Blue: Red

5:1

□ = one part = 10 pens

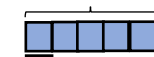


Put back into the question

Blue: Red

(x 10) 5:1 (x 10)  
50:10

Blue pens = 5 x 10 = 50 pens



Red pens = 1 x 10 = 10 pens

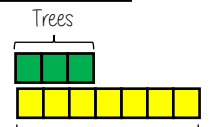
There are 50 Blue Pens

### Ratio as a fraction



Trees: Flowers

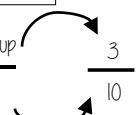
3:7



There are 3 parts for trees

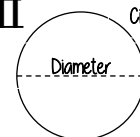
Fraction of trees

Number of parts in group  
Total number of parts



Tree parts 3 + Flower parts 7 = 10

π



Circumference

The ratio of a circle's circumference to its diameter

# YEAR 8 - PROPORTIONAL REASONING...

## Multiplicative Change

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems and explain direct proportion
- Use conversion graphs to make statements, comparisons and form conclusions
- Understand and use scale factors for length

### Keywords

**Proportion:** a statement that links two ratios

**Variable:** a part that the value can be changed

**Axes:** horizontal and vertical lines that a graph is plotted around

**Approximation:** an estimate for a value

**Scale Factor:** the multiple that increases/ decreases a shape in size

**Currency:** the system of money used in a particular country

**Conversion:** the process of changing one variable to another

**Scale:** the comparison of something drawn to its actual size.

### Direct Proportion

As one variable changes the other changes at the same rate.



4 cans of pop = £2.40

4 cans of pop = £2.40  
 $\times 0.5$   
 2 cans of pop = £1.20

This multiplier is the same in the same way that this would be for ratio

This is a multiplicative change

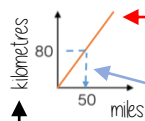
4 cans of pop = £2.40

12 cans of pop = £7.20

Sometimes this is easiest if you work out how much one unit is worth first  
 e.g. 1 can of pop = £0.60

### Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare - then find the associated point by using your graph. Using a ruler helps for accuracy. Showing your conversion lines help as a "check" for solutions

### Conversion between currencies

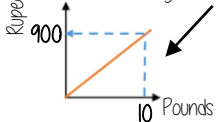


£1 = 90 Rupees

Currency is directly proportional

For every £1 I have 90 Rupees

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

£1 = 90 Rupees  
 $\times 7$   
 £7 = 630 Rupees

### Ratio between similar shapes



Angles in similar shapes do not change. e.g. if a triangle gets bigger the angles can not go above 180°

The two rectangles are similar.



Corresponding sides

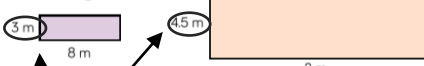
3m : 45m  
 8m : 120m

8m : 12m  
 1m : 15m

Note: Simplify to the same ratio

### Understand Scale Factor

The two rectangles are similar.



$$3 \times 15 = 45$$

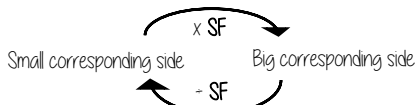
This is a multiplicative change.

Use corresponding sides to calculate a scale factor

Missing length  
 $8 \times 15 = 120$

Scale factor can also be calculated by:

$\frac{\text{Bigger corresponding side}}{\text{Smaller corresponding side}}$



### Draw and interpret scale diagrams

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

Image : Real life  
 1cm : 30cm  
 $\times 10$   
 10cm : 300cm

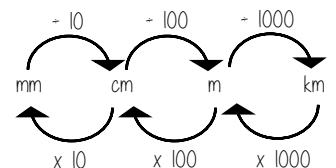


The car in real life is 210cm

Image : Real life  
 1cm : 30cm  
 $\times 7$   
 7cm : 210cm



### Interpret maps with scale factors



1 cm : 250 m

Ratios need to be in the same units

1 cm : 250m

1 cm : 25000cm

$250 \times 100 = 25000$

For every 1cm on my map is 25000cm in real life



# YEAR 8 - PROPORTIONAL REASONING...

## Multiplying and Dividing Fractions

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Carry out any multiplication or division using fractions and integers.
- Solutions can be modelled, described and reasoned.

### Keywords

**Numerator:** the number above the line on a fraction. The top number. Represents how many parts are taken.

**Denominator:** the number below the line on a fraction. The number represent the total number of parts.

**Whole:** a positive number including zero without any decimal or fractional parts.

**Commutative:** an operation is commutative if changing the order does not change the result.

**Unit Fraction:** a fraction where the numerator is one and denominator a positive integer.

**Non-unit Fraction:** a fraction where the numerator is larger than one.

**Dividend:** the amount you want to divide up.

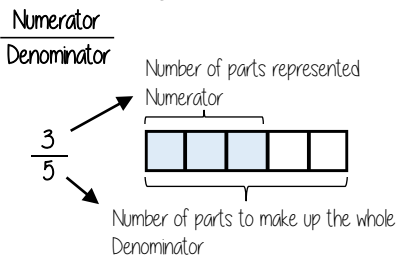
**Divisor:** the number that divides another number.

**Quotient:** the answer after we divide one number by another. e.g. dividend ÷ divisor = quotient

**Reciprocal:** a pair of numbers that multiply together to give 1.

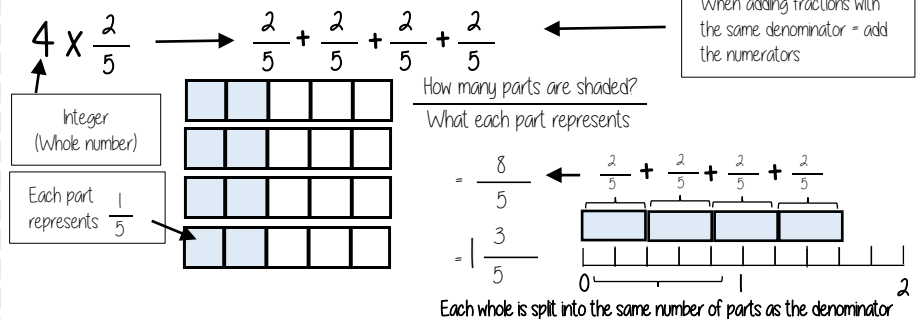


### Representing a fraction

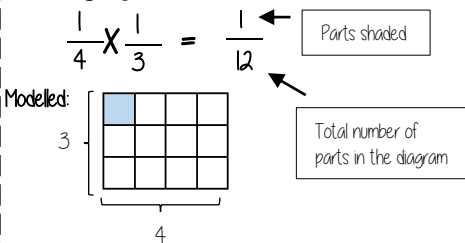


ALL PARTS of a fraction are of equal size

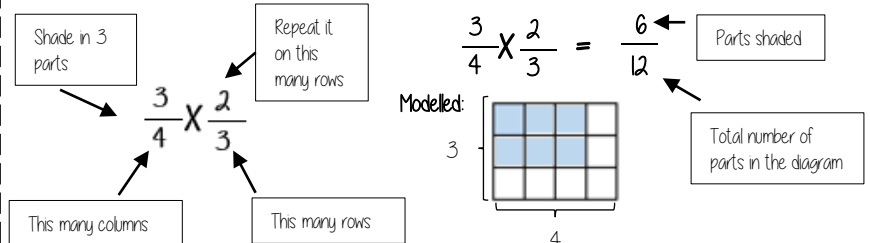
### Repeated addition = multiplication by an integer



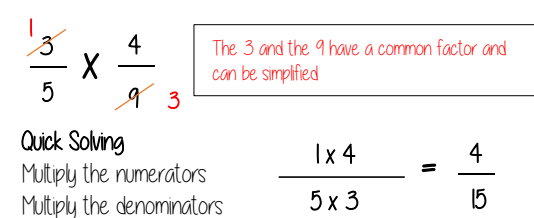
### Multiplying unit fractions



### Multiplying non-unit fractions

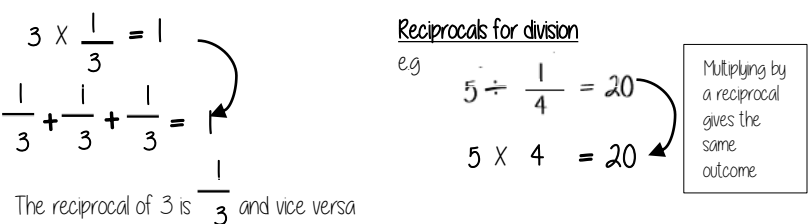


### Quick Multiplying and Cancelling down

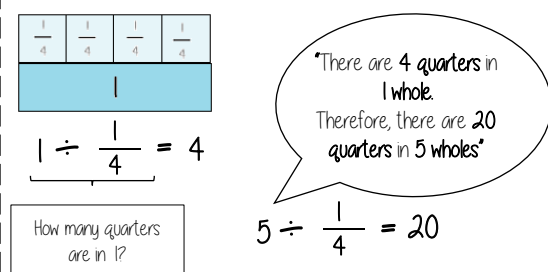


### The reciprocal

When you multiply a number by its reciprocal the answer is always 1

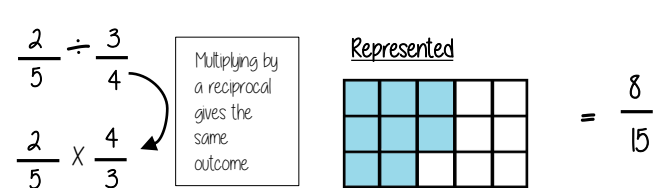


### Dividing an integer by an unit fraction



### Dividing any fractions

Remember to use reciprocals



# YEAR 8 - REPRESENTATIONS...

# Working in the Cartesian plane

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Label and identify lines parallel to the axes
- Recognise and use basic straight lines
- Identify positive and negative gradients
- Link linear graphs to sequences
- Plot  $y = mx + c$  graphs

## Keywords

**Quadrant:** four quarters of the coordinate plane.

**Coordinate:** a set of values that show an exact position.

**Horizontal:** a straight line from left to right (parallel to the x axis)

**Vertical:** a straight line from top to bottom (parallel to the y axis)

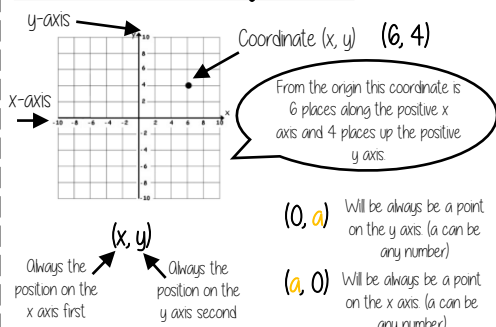
**Origin:** (0,0) on a graph. The point the two axes cross

**Parallel:** Lines that never meet

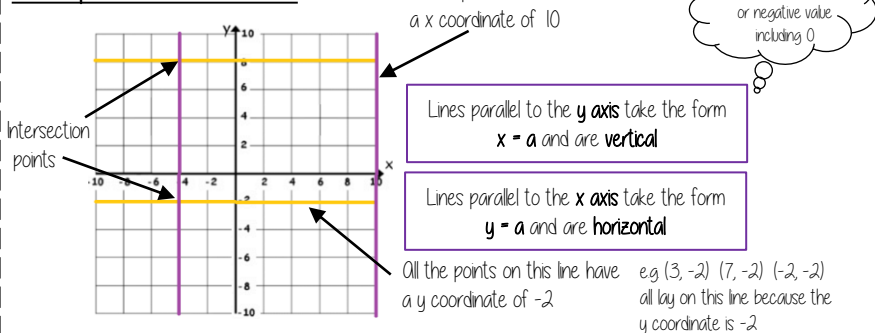
**Gradient:** The steepness of a line

**Intercept:** Where lines cross

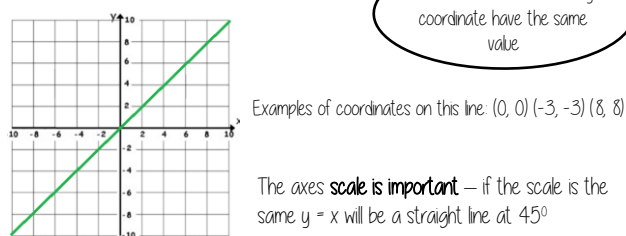
## Coordinates in four quadrants



## Lines parallel to the axes

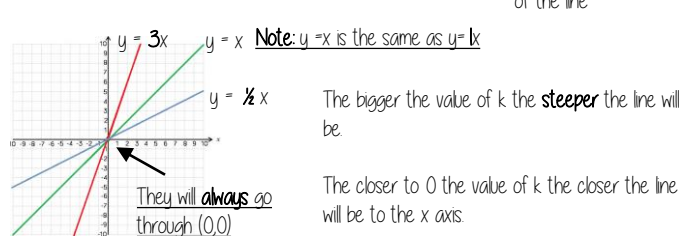


## Recognise and use the line $y=x$

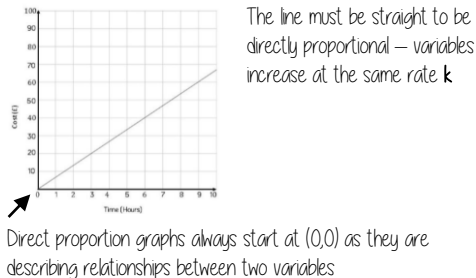


## Recognise and use the lines $y=kx$

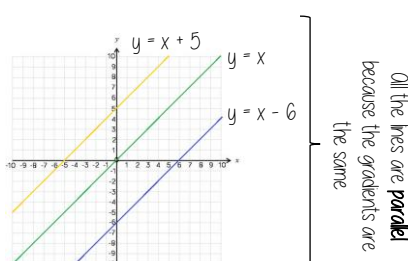
The value of k changes the steepness of the line



## Direct Proportion using $y=kx$



## Lines in the form $y = x + a$

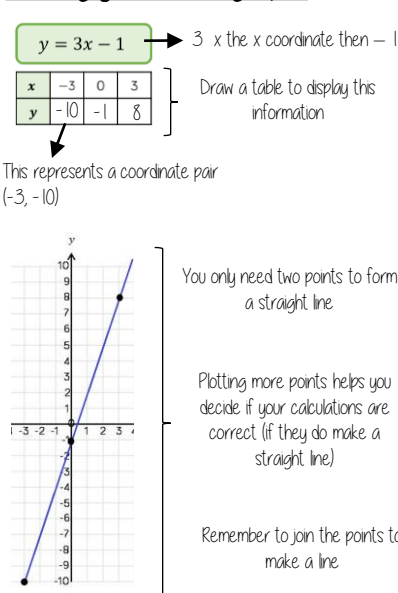


This is the line  $y=x$  when the y and x coordinate are the same

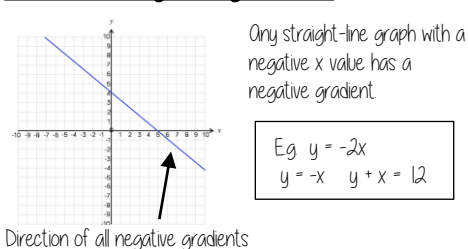
This shows the translation of that line e.g  $y = x + 5$  is the line  $y=x$  moved 5 places up the graph

5 has been added to each of the x coordinates

## Plotting $y = mx + c$ graphs



## Lines with negative gradients



# YEAR 8 - REPRESENTATIONS...

# Representing Data

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

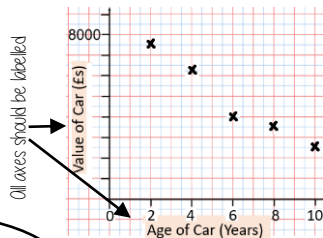
- Draw and interpret scatter graphs
- Describe correlation and relationships
- Identify different types of non-linear relationships
- Design and complete an ungrouped frequency table
- Read and interpret grouped tables (discrete and continuous data)
- Represent data in two way tables

## Keywords

- Variable:** a quantity that may change within the context of the problem
- Relationship:** the link between two variables (items) Eg Between sunny days and ice cream sales
- Correlation:** the mathematical definition for the type of relationship.
- Origin:** where two axes meet on a graph
- Line of best fit:** a straight line on a graph that represents the data on a scatter graph
- Outlier:** a point that lies outside the trend of graph
- Quantitative:** numerical data
- Qualitative:** descriptive information, colours, genders, names, emotions etc
- Continuous:** quantitative data that has an infinite number of possible values within its range
- Discrete:** quantitative or qualitative data that only takes certain values
- Frequency:** the number of times a particular data value occurs

## Draw and interpret a scatter graph

Age of Car (Years)	2	4	6	8	10
Value of Car (Es)	7500	6250	4000	3500	2500



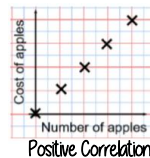
- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship

"This scatter graph shows as the age of a car increases the value decreases"

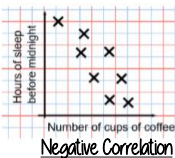
The link between the data can be explained verbally

The axis should fit all the values on and be equally spread out

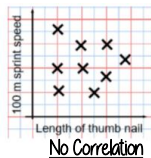
## Linear Correlation



As one variable increases so does the other variable



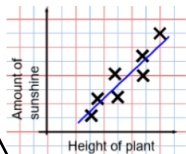
As one variable increases the other variable decreases



There is no relationship between the two variables

## The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph



It is only an estimate because the line is designed to be an average representation of the data

It is always a straight line.

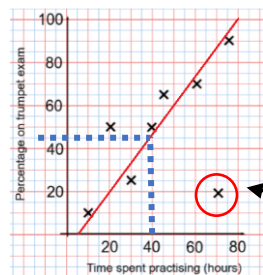
### Things to know:

- The line of best fit **DOES NOT** need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph

## Using a line of best fit

**Interpolation** is using the line of best fit to estimate values inside our data point

e.g 40 hours revising predicts a percentage of 45



**Extrapolation** is where we use our line of best fit to predict information outside of our data

\*\*This is not always useful - in this example you cannot score more than 100%. So revising for longer can not be estimated\*\*

This point is an "outlier" it is an outlier because it doesn't fit this model and stands apart from the data

## Ungrouped Data

The number of times an event happened

The table shows the number of siblings students have. The answers were  
3, 1, 2, 2, 0, 3, 4, 1, 1, 2, 0, 2

2 people had 0 siblings. This means there are 0 siblings to be counted here

Number of siblings	Frequency
0	2
1	3
2	4
3	2
4	1

0 → 0  
3 → 3  
2 + 2 + 2 + 2 OR 2 x 4 = 8  
3 + 3 OR 3 x 2 = 6  
4 → 4

2 people have 3 siblings so there are 6 siblings in total

Best represented by discrete data (Not always a number)

**OVERALL there are 0 + 3 + 8 + 6 + 4 Siblings = 21 siblings**

## Grouped Data

If we have a large spread of data it is better to group it. This is so it is easier to look for a trend. Form groups of equal size to make comparison more valid and spread the groups out from the smallest to the largest value.

Cost of TV (Es)	Tally	Frequency
101 - 150	THH	7
151 - 200	THH THH	11
201 - 250	THH	5
251 - 300		3

**Discrete Data**  
The groups do not overlap

We do not know the exact value of each item in a group - so an estimate would be used to calculate the overall total (Midpoint)

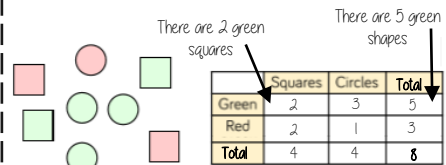
x	Frequency
Weight(g)	
40 < x ≤ 50	1
50 < x ≤ 60	3
60 < x ≤ 70	5

**Continuous Data**  
To make sure all values are included inequalities represent the subgroups

e.g this group includes every weight bigger than 60kg, up to and including 70kg

## Representing data in two-way tables

Two-way tables represent discrete information in a visual way that allows you to make conclusions, find probability or find totals of sub groups



Using your two-way table

To find a fraction  
e.g What fraction of the items are red? **3 red items but 8 items in total = 3/8**

**Interleaving:** Use your fraction, decimal percentage, equivalence knowledge

# YEAR 8 - REPRESENTATIONS... Tables and Probability

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Construct a sample space diagram
- Systematically list outcomes
- Find the probability from two-way tables
- Find the probability from Venn diagrams

## Keywords

**Outcomes:** the result of an event that depends on probability

**Probability:** the chance that something will happen

**Set:** a collection of objects

**Chance:** the likelihood of a particular outcome

**Event:** the outcome of a probability — a set of possible outcomes

**Biased:** a built in error that makes all values wrong by a certain amount

**Union:** Notation 'U' meaning the set made by comparing the elements of two sets

## Construct sample space diagrams



Sample space diagrams provide a systematic way to display outcomes from events

The possible outcomes from tossing a coin

The possible outcomes from rolling a dice

	1	2	3	4	5	6
H	1H	2H	3H	4H	5H	6H
T	1T	2T	3T	4T	5T	6T

This is the set notation to list the outcomes  $S =$

$$S = \{1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T\}$$

In between the  $\{ \}$  are  $a_i$  the possible outcomes

## Probability from sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1H	2H	3H	4H	5H	6H
T	1T	2T	3T	4T	5T	6T

This is the set notation that represents the question P

What is the probability that an outcome has an even number and a tails?

$$P(\text{Even number and Tails}) = \frac{3}{12}$$

In between the  $( )$  is the event asked for

There are three even numbers with tails

Numerator: the event

Denominator: the total number of outcomes

There are twelve possible outcomes

## Probability from two-way tables

	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

$$P(\text{Girl walk to school}) = \frac{21}{100}$$

The total number of items

The event

The total in the set

## Product Rule

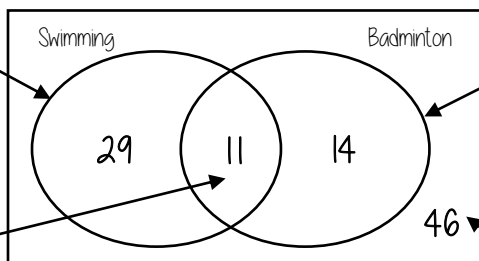
The number of items in event a

x

The number of items in event b

## Probability from Venn diagrams

This whole curve includes everyone that went swimming. Because 11 did both we calculate just swimming by  $40 - 11$



The intersection represents both Swimming AND badminton

This whole curve includes everyone that went to badminton. Because 11 did both we calculate just badminton by  $25 - 11$

The number outside represents those that did neither badminton or swimming

$$P(\text{Just swimming}) = \frac{29}{100}$$

$$100 - 29 - 11 - 14$$

100 students were questioned if they played badminton or went to swimming club. 40 went swimming, 25 went to badminton and 11 went to both