

ANGLES (Unit 6 Foundation)

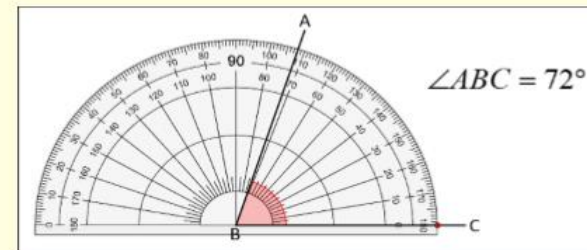
Angles at a point add up to 360° .	 $a + b + c + d = 360^\circ$
Angles on a straight line add up to 180° .	 $a + b + c = 180^\circ$
The interior angles in any triangle add up to 180° .	 $a + b + c = 180^\circ$
The interior angles in an equilateral triangle are all 60° .	

Angle	Vertically opposite
Alternate	Perpendicular
Supplementary Co-interior	Parallel lines
Acute/Obtuse/ Reflex	corresponding

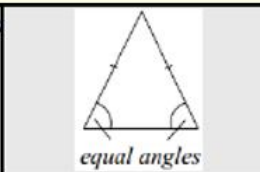
Videos	
Names of angles	V38
Angles in a triangle	v37
Angles on a line/ around a point	V35 V30
Angles and parallel lines	V25
Properties of special triangles	V327

Key facts to memorise- polygon angle facts

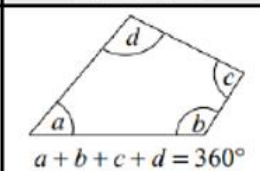
Polygon names		Polygon angle facts	
3 sides	Triangle	Sum of interior angles in a polygon with n sides = $(n - 2) \times 180$	
4 sides	Quadrilateral		
5 sides	Pentagon		
6 sides	Hexagon	Sum of exterior angles in a polygon = 360°	
7 sides	Heptagon		
8 sides	Octagon	Interior angle + exterior angle = 180°	
9 sides	Nonagon		
10 sides	Decagon		



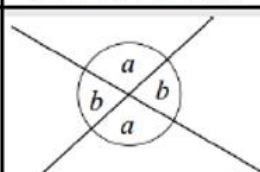
An isosceles triangle has two angles of the same size.



The interior angles in any quadrilateral add up to 360° .



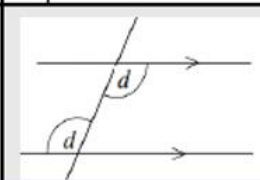
When two straight lines intersect, the opposite angles are equal.



When a straight line intersects a pair of parallel lines, the **corresponding angles** are equal.



When a straight line intersects a pair of parallel lines, the **alternate angles** are equal.



Mean, Median, Mode and Range

MEAN

$$\frac{\text{Sum of all values}}{\text{Number of values}}$$

MEDIAN

Middle value when numbers are placed in order

MODE

Most Common

RANGE

Largest value – smallest value

Practise

Find the mean, median, mode and range of the following set of numbers

1, 3, 2, 8, 7, 9, 5, 4, 10, 2, 4,

Challenge Question

12	6	15	?
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The mean of these 4 cards is 10, what is the missing number?

Mean from frequency table video

Averages and Range (Unit 7 Foundation)

Averages and Range from a Frequency table

20 students scored goals for the school hockey team last month. The table gives information about the number of goals they scored.

Goals scored	Number of students	Goals scored x no. of students
1	9	1 x 9 = 9
2	3	2 x 3 = 6
3	5	3 x 5 = 15
4	3	4 x 3 = 12
20 students		42 goals scored

This means 9 students each scored 1 goal

Add a totals row to work out the total number of students and total goals scored

Add an extra column to work out the number of goals scored

Mean = total number of goals scored divided by the number of students
 = $42 \div 20 = 2.1$ goals per student

Mode = most common number of goals scored
 = 1 (as 9 students scored 1 goal which is more than any other number of goals)

Median = the number in the middle = 2
 If I wrote the goals scored by each student as a list it would look like:

1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4

9 students scored 1 goal each 3 students scored 4 goals each

The **median** is the middle number now that they are in order

Range of the number of goals scored = $4 - 1 = 3$

video

Stem and leaf Diagrams



This stem and leaf diagram shows the times, in seconds, for a group of swimmers to swim 100m. Find the median and the mode.

55	2	3	6
56	3	3	7
57	0	2	6
58	4	4	5
59	3		

Count the number of values; 17.
 The median is the $\frac{n+1}{2}$ th value. There are 17 values, so $n = 17$.
 $\frac{17+1}{2} = 9$
 In a stem and leaf diagram the data is in order. So count up to the 9th value.
 Look for repeated values in the rows. 57 | 0 2 6 6 6 7
 The mode is 57.6 seconds.

Estimating the mean from a grouped frequency table [Estimated mean video](#)

The table represents the scores of 30 students in a maths test

Score	Frequency	Midpoint of class, m	m x f
1-5	5	3	15
6-10	6	8	48
11-15	9	13	117
16-20	10	18	180
Total	30		360

Estimate of mean = $\frac{360}{30} = 12$

This is only an **ESTIMATE** for the mean as we are estimating the scores of the students by using the mid-point

Practise Question

Real In a survey, 30 small companies were asked how many employees they had. This table shows the results.

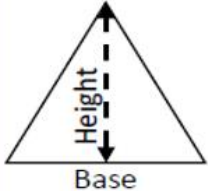
Number of employees	Frequency, f	Midpoint of class, m	m x f
1-5	12		
6-10	7		
11-15	6		
16-20	5		
Total			Total

Calculate an estimate for the mean number of employees per company.

Area

Is the inside of a shape.

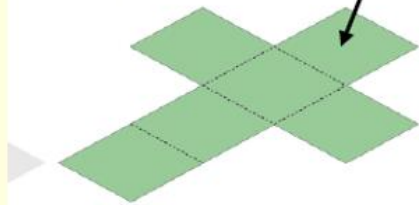
Area of **Rectangle** = length \times width



Area of **Triangle** = $\frac{1}{2} \times \text{base} \times \text{height}$

Surface Area

The area of each face added together.



Circles

Key Facts – Circle Formulae

$$A = \pi r^2$$

$$A = \frac{\pi d^2}{4}$$

Used to calculate the area of a circle. Notice that the formula includes a a^2 and the answer will be an area measured in units². This formula can also be used as the basis for finding the area of sectors and the volumes of cylinders.

$$C = 2\pi r$$

$$C = \pi d$$

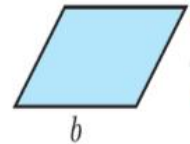
Used to calculate the circumference. Notice that the formula does not feature a a^2 . This formula can also be used to calculate the perimeter of shapes made up from part of a circle.

Perimeter, Area and Volume

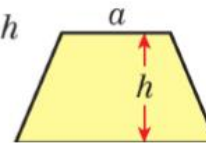
(Unit 8 Foundation)

VIDEOS: [V44](#) [V45](#) [V49](#) [V40](#) [V355](#)

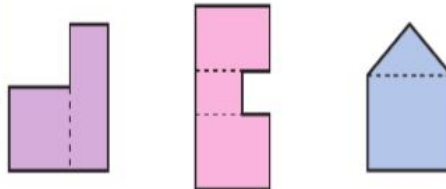
Area of a parallelogram = base \times vertical height
 $= b \times h$



Area of trapezium = $\frac{1}{2} \times (a + b) \times h$



To find the area of a **compound shape**, draw lines to split the shape into simple shapes. Find the area of each shape separately. Add to find the total area.



The **volume** of a 3D solid is the amount of space it takes up. Volume is measured in mm³, cm³ or m³.

Volume of a prism = area of cross-section \times length

Perimeter

Is the distance round the edges of the shape

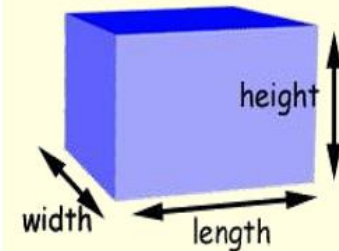


5cm

$$P = 5 + 4 + 5 + 4 = 18\text{cm}$$

Volume

volume = length \times width \times height



Graphs (Unit 9 Foundation)



Co-ordinates

These are given in the form (X,Y). We go along the x axis and up or down the y axis.

Y intercept

This is the point where the line crosses the y axis. On the example the y intercept = +2

Gradient

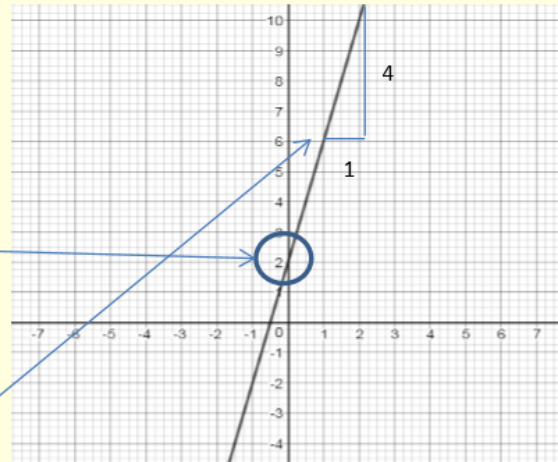
The steepness of a graph is called the **GRADIENT**. You can find the gradient by :

Squares up or down
Squares across

$$\frac{4}{1}$$

Gradient + 4

Gradient can be positive (/) or negative (\)



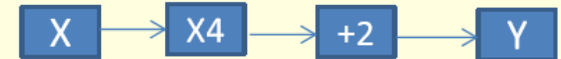
Parallel Lines have the same gradient but a different y intercept. For example a parallel line for the above graph would be $y = 4x - 3$

Mid points is the point exactly in the middle. To find the coordinates add the x coordinates together and divide by 2 and do the same for the y coordinates.

Table of Values/ Plotting graphs

To find the coordinates of a straight line you can use a table of values.

Firstly create a function machine



Then input numbers from the x axis to find the y axis.

These create coordinates which you can then plot onto the graph and join up with a ruler.

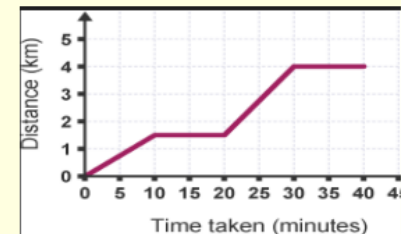
X	0	1	2	3
Y	2	6	10	14

Distance time graphs

Represents a journey. The vertical axis represents the distance from starting point. The horizontal line represents time taken.

A horizontal line on a distance time graph represents an object at rest.

The gradient of the line represents the speed of the journey



$$Y = mx + c$$

Gradient

Y intercept

You can use the gradient and y intercept to write an equation for a line.
Equation for above line is $y = 4x + 2$

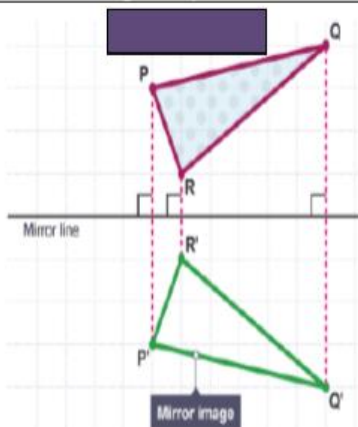
Transformations (Unit 10 Foundation)

Reflection

Every point in the image is the same distance from the mirror line as the original shape.

The line joining a point on the original shape to the same point on the image is perpendicular to the mirror line.

A reflection creates a congruent image



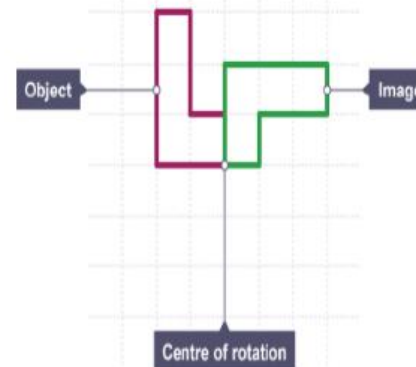
Rotation

Rotation turns a shape around a fixed point called the **centre of rotation**.

3 parts of a rotation

- the centre of rotation
- the angle of rotation
- the direction of rotation

A Rotation creates a congruent image



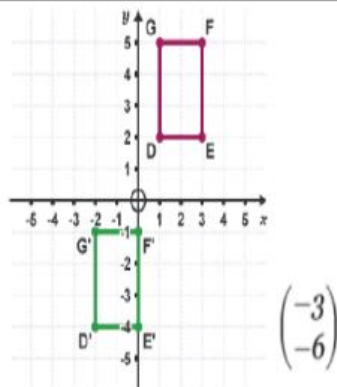
Translation

A **translation** moves a shape up, down or from side to side and creates a congruent image.

Column vectors are used to describe translations

$\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ means translate the shape 4 squares to the right and 3 squares down.

$\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ means translate the shape 2 squares to the left and 1 square up.



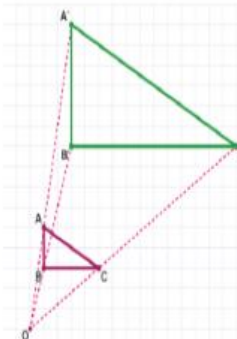
Enlargement

Enlarging a shape changes its size

2 parts of an enlargement

- the scale factor
 - the centre of enlargement
- Fractional SF reduces the shape
Negative SF inverts the shape

An enlargement creates a similar shape



ABC has been enlarged by sf 3 about O.

Linked Prior Topics

Shapes
Scales
Angles
Straight line graphs

Vocabulary

Object – Starting shape
Image – Created by a transformation
Congruent – 2 shapes are exactly the same
Similar – 2 shapes with the same angles but different length sides
Perpendicular – Forms a 90° angle

Linked Future Topics

Transformation of functions
Similar shapes

RATIO

This is used to compare two or more amounts
Always draw boxes when dealing with ratio!

Writing a Ratio

The amount of one object compared with another. Eg there are 2 triangles to 5 squares

2:5



Simplifying a Ratio [Video 269](#)

You simplify a ratio by dividing the numbers by the HCF (Highest Common Factor)

Simplify 6:12

Divide both by 6

1:2

Simplify 3:9:15

Divide all numbers by 3

1:3:5

Simplify 6:1.5

Multiply both sides by 2

12:3

Divide both sides by 3

4:1

Unit 11 Foundation Ratio & Proportion

Sharing an Amount in a Ratio

[Video 270](#)

Mr Musson and Mr Coren get £72 pocket money. They share it in the ratio 5:3.

Draw a total of 8 boxes ($5 + 3 = 8$)
Split the money evenly between each box ($72 \div 8 = 9$)

Mr Musson gets 5 boxes = $5 \times 9 = £45$
Mr Coren gets 3 boxes = $3 \times 9 = £27$

£45 for Mr Musson £27 for Mr Coren

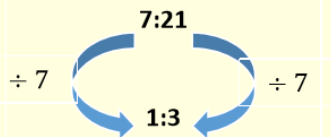


£72 in total

Writing in the Ratio 1:n

You need to divide **both** sides by the **same** amount until the correct number is down to 1

Write **7:21** in the ratio **1:n**



PROPORTION

Proportion compares a part with a whole

[Video 210](#)

Best Buy

Video

This is about finding which item is better **value for money**

Example 1



A pack of 4 tins of baked beans cost £1.96



A pack of 6 tins of baked beans cost £3

*Hint: Find the cost of **one** tin from each pack*

$$\begin{aligned} £1.96 \div 4 &= £0.49 \\ &= 49\text{p per tin} \end{aligned}$$

$$\begin{aligned} £3 \div 6 &= £0.50 \\ &= 50\text{p per tin} \end{aligned}$$

Therefore the pack of 4 tins is better value for money

Example 2

Radox hand wash is on sale at Boots and Superdrug

Boots
500ml bottle costs £2.24

Superdrug
200ml bottle costs 90p

*Hint: multiply both to the **same** amount of hand wash*

$$\begin{aligned} \times 2 \quad 500\text{ml} &= £2.24 \\ \quad \quad \quad 1000\text{ml} &= £4.48 \end{aligned}$$

$$\begin{aligned} \times 5 \quad 200\text{ml} &= 90\text{p} \\ \quad \quad \quad 1000\text{ml} &= £4.50 \end{aligned}$$

Therefore the bottle from boots is better value for money

This is an example of DIRECT PROPORTION

Pythagoras Theorem

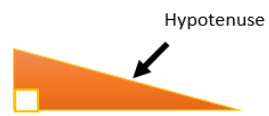
$$a^2 + b^2 = c^2$$

Pythagoras is used to find missing sides in **Right-angled triangles**

Key Facts

HYPOTENUSE

This is the longest side in a right-angled triangle and is **ALWAYS** opposite the right angle



Method to find the hypotenuse

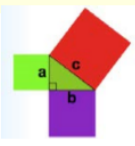
[Pythagoras video 257](#)

- Square side a
- Square side b
- Add together
- Square root

$$a^2 + b^2 = c^2$$

Method to find a shorter side

- Square side c
- Square side a/b (whichever is known)
- Subtract a/b from c
- Square root



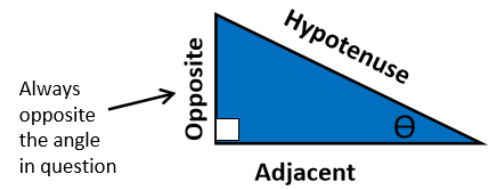
Unit 12 Foundation Right-Angled Triangles 1

[Trigonometry Video 329, 330, 331](#)

Trigonometry

Used to find missing sides and angles in right-angled triangles

You must label your sides correctly



Using the Tangent Ratio:

$$Opp = \tan \theta \times Adj$$

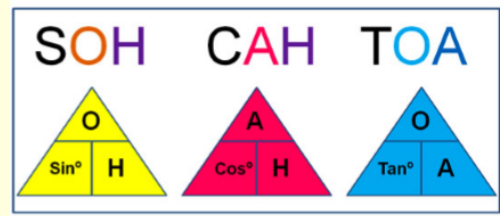
$$Adj = \frac{Opp}{\tan \theta}$$

$$\tan^{-1} \theta = \frac{Opp}{Adj}$$

Press shift and then tan on your calculator

SOH – CAH – TOA Pyramids

Cover the letter which is the unknown value, and then Multiply for horizontal relationships and Divide for vertical relationships



Using the Sine Ratio

$$Opp = \sin \theta \times Hyp$$

$$Hyp = \frac{Opp}{\sin \theta}$$

$$\sin^{-1} \theta = \frac{Opp}{Hyp}$$

Press shift and then sin on your calculator

Using the Cosine Ratio:

$$Adj = \cos \theta \times Hyp$$

$$Hyp = \frac{Adj}{\cos \theta}$$

$$\cos^{-1} \theta = \frac{Adj}{Hyp}$$

Press shift and then cos on your calculator

The **probability** of something (let's call it outcome **A**) happening is written as **P(A)**, and must be between 0 and 1.

If **P(A) = 0**, it is **impossible**.

If **P(A) = 1**, it is **certain** to happen.



The probability of A **not** happening is written as **P(A')**. Since A will either happen or not happen,

P(A) + P(A') = 1

[Video 250: Events not happening](#)

We call the two outcomes above "**mutually exclusive**" – this means they cannot happen at the same time. The probabilities of all possible outcomes for an event always add up to 1, because one of them is certain to happen.

Unit 13 Foundation

Example:

Event: rolling **3** on a fair six-sided dice.

$P(3) = 1/6$

$P(3') = 5/6$

These two outcomes are **mutually exclusive** and cover every possibility, so their probabilities **add up to 1**

[Video 249: Independent Events](#)

If the outcome of one event doesn't affect the outcome of another, we call those events **independent**. For example, **flipping a coin** and **rolling a dice** are independent of each other.

Experimental probability is about **estimating** probability based on **previous outcomes**, (unlike **theoretical** probability, which was used above and is based on **what should happen**). Experimental probability would be written as

[Video 248: Relative Frequency](#)

$$\frac{\text{frequency of desired outcomes}}{\text{total number of trials}}$$

"Trials" refers to **what you actually do** for your experiment (flipping a coin; counting cars as they drive past). Each time you do it counts as one trial.

Example:

Experiment: spinning a fair four-numbered spinner 100 times (i.e. 100 trials)

Score	1	2	3	4
Frequency	23	26	30	21

Based on these results, the **P(1) = 23/100**, or **0.23**. To estimate **relative frequency**, multiply the number of intended trials by the experimental probability, e.g. for **200 trials**, we would predict **46** results will be 1 because **200 x 0.23 = 46**, and **60** results will be 3 because **200 x 0.30 = 60**.

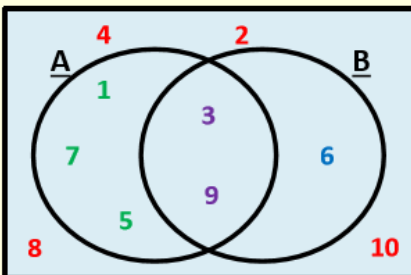
Venn diagrams show how two or more **sets** (groups) can overlap, and we can use them to calculate the probability of a given **element** (item in a set) being chosen. They can have each element individually written in them (**Example 1**), or just the quantity of each section (**Example 2**). The **universal set** (ξ) contains **everything** being considered.

Example 1

ξ : Integers up to 10

A: Odd numbers

B: Multiples of 3



$A \cap B$: **only** elements in both **A** and **B**

$A \cup B$: **all** elements in **A** or **B** or both

If picking a number at random,

$P(A \cap B') = 3/10$

$P(A' \cap B) = 1/10$

$P(A \cap B) = 2/10$

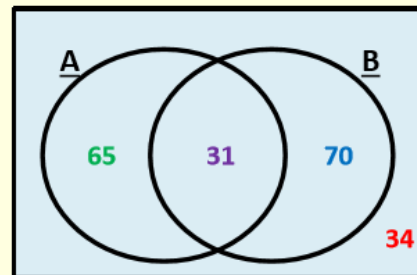
$P(A' \cap B') = 4/10$

Example 2

ξ : Year 10 students (200 in total)

A: Students who walk to school

B: Students who like football



$A \cap B'$: elements in **A** and **not** in **B**

$A' \cap B$: elements in **B** and **not** in **A**

[Video 380: Venn Diagrams](#)

If picking a student at random,

$P(A \cap B') = 65/200$

$P(A' \cap B) = 70/200$

$P(A \cap B) = 31/200$

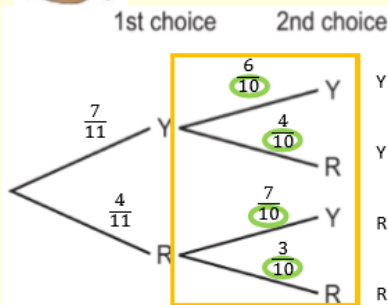
$P(A' \cap B') = 34/200$

Probability "tree" diagrams show the possible outcomes of multiple events one after the other. The "branches" are for each outcome and every set of branches adds to 1.



There are 11 balls in the bag, so the first choice is out of 11. The second choice is out of 10, since a ball has been taken out, so the denominators change

The second choice has two sets of branches because there are two possible scenarios for it (either after a yellow or after a red).



Y then Y: $P(YY) = \frac{7}{11} \times \frac{6}{10} = \frac{42}{110}$

Y then R: $P(YR) = \frac{7}{11} \times \frac{4}{10} = \frac{28}{110}$

R then Y: $P(RY) = \frac{4}{11} \times \frac{7}{10} = \frac{28}{110}$

R then R: $P(RR) = \frac{4}{11} \times \frac{3}{10} = \frac{12}{110}$

To find the final probability, multiply along the branches as shown.

(For example, the probability of picking red both times is **12/110**.)

[Video 252: Tree Diagrams](#)