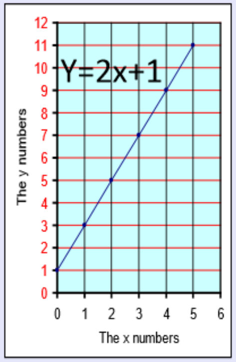
Unit 6 Higher Graphs – Links: V191 V171 V197 V196

Year 10 – Maths Higher / Unit 6 Graphs

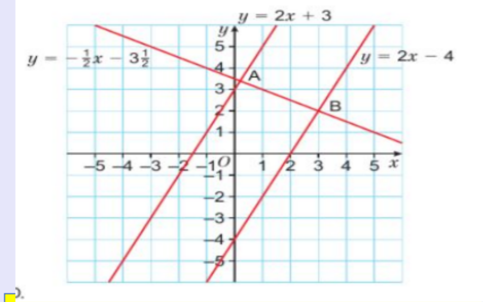
Linear Equations
Y = mx + c

where m is the gradient

C is where the graph crosses the y axis

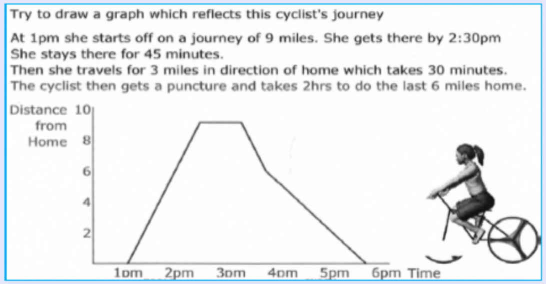


Parallel lines have same gradient



A distance – time graph represents a journey

The gradient is the speed



Try to draw a graph which reflects this cyclist's journey

At 1pm she starts off on a journey of 9 miles. She gets there by 2:30pm
She stays there for 45 minutes.
Then she travels for 3 miles in direction of home which takes 30 minutes.
The cyclist then gets a puncture and takes 2hrs to do the last 6 miles home.

Perpendicular lines have gradients that multiply to give -1

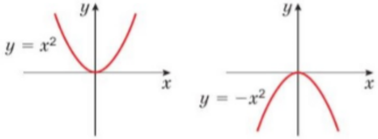
When a graph has gradient m, the perpendicular line to that will have gradient - $\frac{1}{m}$

Velocity- time graph
Straight line – means constant acceleration

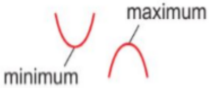
Direct proportion is shown by a straight line graph through the origin

The equation of a circle with centre (0,0) and radius r is $x^2 + y^2 = r^2$

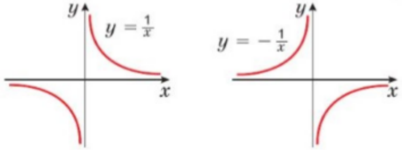
A **quadratic equation** contains a term in x^2 but no higher power of x. The graph of a quadratic equation is a curved shape called a **parabola**.



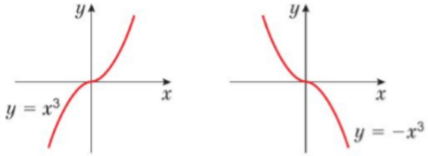
A quadratic graph has either a **minimum point** or a **maximum point** where the graph turns.



Reciprocal functions are in the form $\frac{k}{x}$ where k is a number.



A **cubic function** contains a term in x^3 but no higher power of x. It can also have terms in x^2 and x and number terms.



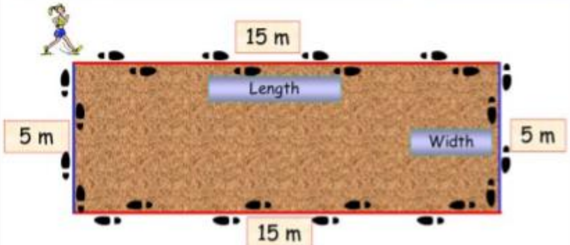
Knowledge Organiser: Unit 7 Higher (Area and Volume)

Corbett Maths video links: V312 V377 V358

Perimeter

The perimeter of a shape is the distance around the outside.

Rectangles



Perimeter = 15 m + 5 m + 15 m + 5 m = 40 m

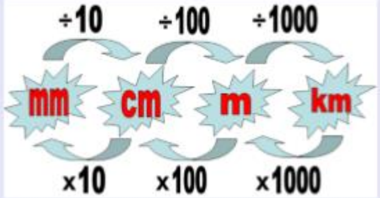
The Area of a Circle

Find the area of the $\frac{1}{4}$ and $\frac{3}{4}$ circles.

$A = \frac{1}{4}\pi r^2$
$= \frac{1}{4} \times \pi \times 6^2$
$=$ 28.3 cm² (1 dp)

$A = \frac{3}{4}\pi r^2$
$= \frac{3}{4} \times \pi \times 8.5^2$
$=$ 170.2 cm² (1 dp)

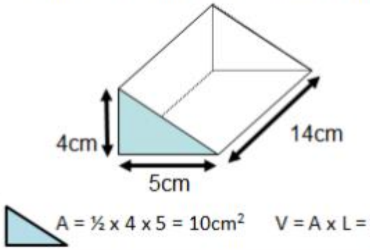
Metric conversions:



VOLUME is how many cubic units fit **inside** a shape.

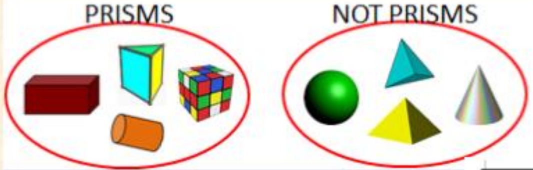
For a prism* Volume = Area x length

*a shape that is the same all the way along its length

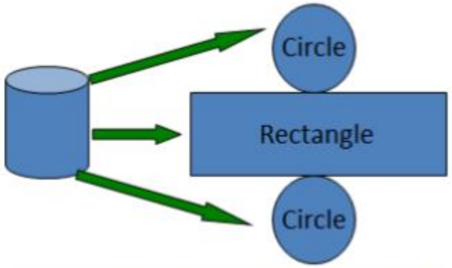


A = ½ x 4 x 5 = 10cm² V = A x L = 10 x 14 = 140cm³

So, always start by working out the **area** on front of the shape – this has to be the same all the way along the length (i.e. it has to be a prism).

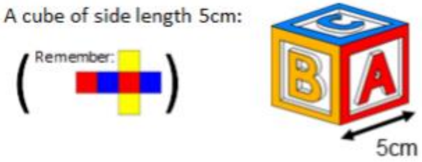


SURFACE AREA is how many square units fit onto the **outside** of a shape.



It's helpful to think of the net of the shape: the surface area is just the area of all the bits of the net added together.

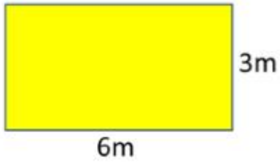
e.g. A cube of side length 5cm:



Area of one face = 5 x 5 = 25cm²

Total surface area = 25 x 6 = 150cm²

Error bounds:



The lengths have been measured to the nearest metre

What the minimum and maximum values that the base and height could be?

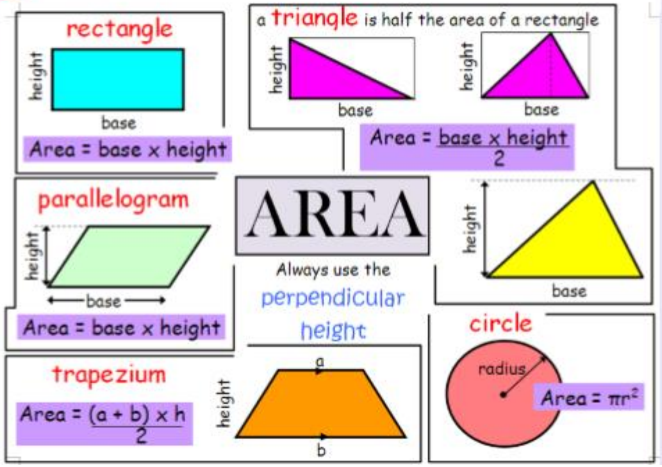
5.5 ≤ base < 6.5m 2.5 ≤ height < 3.5m

What the minimum and maximum values that the *perimeter* could be?

16m ≤ perimeter < 20m

What the minimum and maximum values that the *area* could be?

13.75m² ≤ area < 22.75m²



Area = base x height

Area = $\frac{\text{base x height}}{2}$

Area = base x height

Always use the perpendicular height

Area = $\frac{(a + b) \times h}{2}$

Area = πr^2

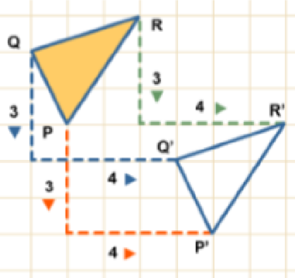
Year 10 – Maths Higher / Unit 7 Area and Volume

KS4 Knowledge Organiser

Higher Tier Unit 8: Transformations & Constructions

Translation: V325

To translate means to move a shape. The shape does not change size or orientation.



Column Vector:

In a column vector, the top number moves left (-) or right (+) and the bottom number moves up (+) or down (-)

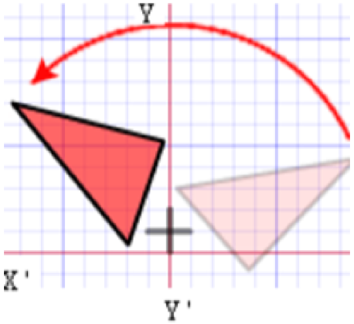
$\begin{pmatrix}2\\3\end{pmatrix}$ means '2 right, 3 up'

$\begin{pmatrix}-1\\-5\end{pmatrix}$ means '1 left, 5 down'

Rotation: V275

The size does not change, but the shape is turned around a point. (Use tracing paper).

Rotate the triangle 90° anti-clockwise about (0,1).

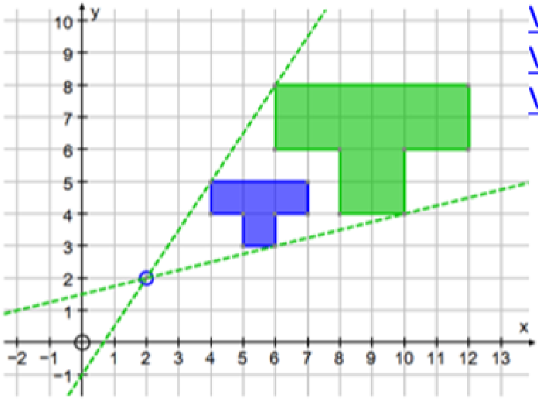


Enlargement:

The shape will get **bigger** or **smaller**. Multiply each side by the **scale factor**.

Scale Factor = 3 means '3 times larger = multiply by 3'

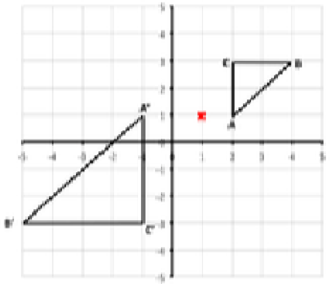
Scale Factor = ½ means 'half the size = divide by 2'



V104
V105
V106

V107 V108

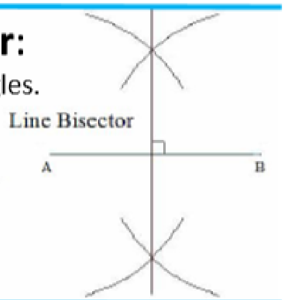
Negative Scale Factor Enlargements will look like they have been rotated. $SF = -2$ will be rotated. & also twice as big. Enlarge ABC by scale factor -2, centre (1,1)



Perpendicular Bisector:

Cuts a line in half and at right angles.

V78



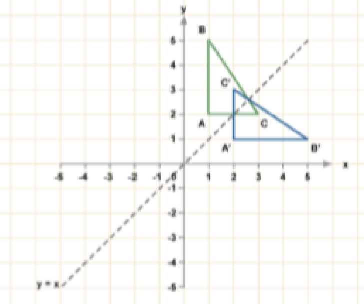
Reflection:

The size does not change, but the shape is '**flipped**' like in a **mirror**.

Line **x=?** is a **vertical line.**
Line **y=?** is a **horizontal line.**
Line **y=x** is a **diagonal line.**

V272 V273 V274

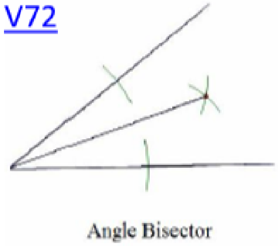
Reflect shape C in the line y=x



Angle Bisector:

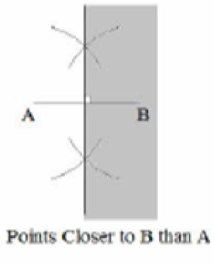
Cuts the angle in half.

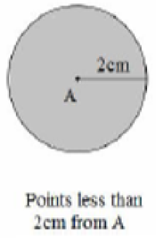
V72

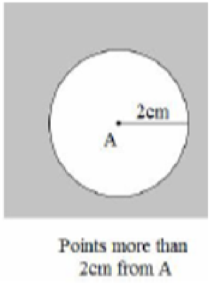


Loci: A locus is a path of points that follow a rule.

V75 V76 V77







KS4 Knowledge Organiser

Higher Tier Unit 9: Equations & Inequalities

Quadratic: V325

A quadratic expression is of the form **$ax^2 + bx + c$** where a, b and c are numbers, **$a \neq 0$**

Examples of quadratic expressions: **x^2 or $8x^2 - 3x + 7$**

Factorising Quadratics: V118 V119

When a quadratic expression is in the form $x^2 + bx + c$ find the 2 numbers that add to give b & multiply to give c.

e.g. $x^2 + 7x + 10 = (x+5)(x+2)$

(because 5 and 2 add to give 7 and multiply to give 10)

Difference of Two Squares V120

An expression of the form a^2-b^2 can be factorised to give (a+b)(a-b).

e.g. $x^2 – 25 = (x+5)(x-5)$ or $16x^2 – 81 = (4x+9)(4x-9)$

Solving Quadratics ($ax^2 = b$)

Isolate the x^2 term and square root both sides.

e.g. $2x^2 = 98$
$x^2 = 49$
$x = \pm 7$

Remember there will be a positive and a negative solution.

Solving Quadratics ($ax^2 + bx = 0$) V266

Factorise and then **solve = 0**

e.g. $x^2 - 3x = 0$
$x(x-3) = 0$
$x = 0$ or $x = 3$

e.g. Solve $x^2 + 3x – 10 = 0$
Factorise: $(x+5)(x-2) = 0$
$x=-5$ or $x=2$

Simultaneous Equations:

A set of two or more equations, each involving two or more variables (letters).

The solutions to simultaneous equations satisfy both/all of the equations.

V295 V296 V297

e.g. $2x + y = 7$
$3x – y = 8$

$x=3$, $y=1$

Factorising Quadratics when a ≠ 1 V266

When a quadratic is in the form **$ax^2 + bx + c$**

1. Multiply a by c = ac
2. Find two numbers that add to give b and multiply to give ac.
3. Re-write the quadratic, replacing bx with the two numbers you found.
4. Factorise in pairs – you should get the same bracket twice
5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.

Completing the Square V267a V371

A quadratic in the form **$ax^2 + bx + c$** can be written in the form **$(x + p)^2 + q$**

1. Write a set of brackets with x in and half the value of b.
2. Square the bracket.
3. Subtract $(b/2)^2$ and add c.
4. Simplify the expression.

Solving Quadratics using the Quadratic Formula: V267

A quadratic in the form **$ax^2 + bx + c$** can be solved using the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the formula if the quadratic does not factorise easily.

Inequality symbols: V176 V177 V178 V179

x>2 means x is **greater than** 2
x<3 means x is **less than** 3
x≥1 means x is **greater than or equal to** 1
x≤6 means x is **less than or equal to** 6

Inequalities can be shown on a number line.

Open circles are used for numbers that are **less than or greater than** (< or >)

Closed circles are used for numbers that are **less than or equal or greater than or equal** (≤ or ≥)

$x \geq 0$

$x < 2$

$-5 \leq x < 4$

Year 10 – Maths Higher / Unit 9 Equations & Inequalities

Unit 10 Higher (Probability)

Corbett Maths video links: V244 V250 V247

TECHNICAL LANGUAGE:

P("something") means *probability of "something" happening*

"Mutually exclusive" means that if one thing happens, the other cannot. E.g. being alive and dead are mutually exclusive states!

"Bias" = unfairness. It would be biased to roll a die that has 2 sixes on it and no zeroes in a normal dice game.

If outcomes A and B are mutually exclusive,
P(A) + P(B) = 1 or 1-P(A) = P(B)

E.g. If there is no draw allowed,
and P(win) = 0.7, B(lose) must be 0.3

Sometimes bias is difficult to spot in experiments.
If you flip a coin 100 times, you expect 50 heads
and 50 tails, but does that mean your coin is biased
if you get 60:40? What about 90:10?? What about 99:1????

On fair dice, opposite faces *should* add up to 7.

COMBINING PROBABILITIES:

If you want to find the probability of 2 things happening, MULTIPLY the individual probabilities.

One of the reasons why fractions are convenient for probability is That they are so easy to multiply; ½ x ⅝ = 5/16

Multiply numerators, multiply denominators

Remember to simplify whenever possible

Example:

P(win) = 2/5   P(win) = 3/10   P(win both) = 2/5 x 3/10 = 6/50 = 3/25

The LANGUAGE of probability:

P("something") means *probability of "something" happening*

Eg. When tossing a coin P(heads) = 0.5 or ½
P(tails) = 0.5 or ½
P(heads or tails) = 1 (certain)
P(coin flying off into outer space) = 0 (impossible)

It's often easiest to write probabilities as **fractions***, especially if you want to combine probabilities in tree diagrams...

$$* \ \frac{\text{how many ways it can happen}}{\text{How many outcomes there are altogether}}$$

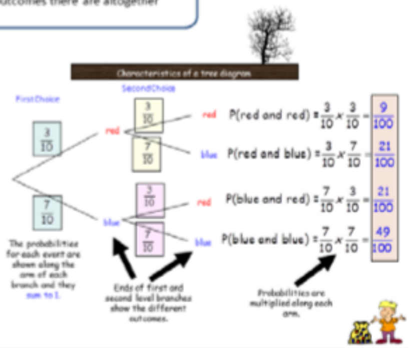
Sample Space Diagrams:
Often used to find all the possible combinations of 2 events being combined:

Roll a die

Roll a die

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

If we're adding, The value in the (6,6) box of the SSD would be 12



You can use two-way tables to help solve probability problems:

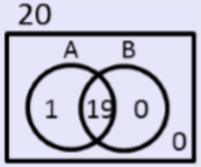
	France	Holland	Elsewhere	Total
June	6	18	5	29
July	10	19	2	31
August	15	15	10	40
Total	31	52	17	100

What is the probability that a person selected at random:

1. Went to Holland on holiday?	52/100
2. Went on holiday in July?	31/100
3. Went to France in August?	15/100
4. Did not visit either France or Holland?	17/100
5. Went on holiday in June?	29/100

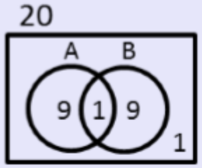
VENN DIAGRAMS

$$P(A \cap B) = \frac{19}{20}$$



20 people chose A, and 19 chose B.

$$P(A \cup B) = \frac{19}{20}$$



1 person opts out of choosing either A or B.

Year 10 – Maths Higher / Unit 10 Probability

Year 10 – Maths Higher / Unit 11 Multiplicative Reasoning

Unit 11 Multiplicative Reasoning V236 V384 V385 V254 V255

Exmouth Community College Academy Trust

In **compound interest** the interest earned each year is added to money in the account and earns interest the next year.
Most interest rates are compound interest rates.

If y is directly proportional to x, $y \propto x$ and $y = kx$, where k is a number, called the **constant of proportionality**.

Where k is the constant of proportionality:

- if y is proportional to the square of x then $y \propto x^2$ and $y = kx^2$
- if y is proportional to the cube of x then $y \propto x^3$ and $y = kx^3$
- if y is proportional to the square root of x then $y \propto \sqrt{x}$ and $y = k\sqrt{x}$

These are three kinematics formulae:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken

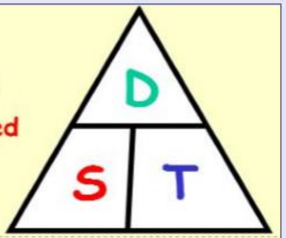
You can calculate an amount after n years' compound interest using the formula

$$\text{amount} = \text{initial amount} \times \left(\frac{100 + \text{interest rate}}{100}\right)^n$$

Multiplicative means involving multiplication or division

Key Words
Velocity
Acceleration
Force
Pressure

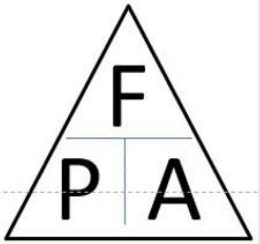
Distance = Time X Speed
Speed = Distance ÷ Time
Time = Distance ÷ Speed



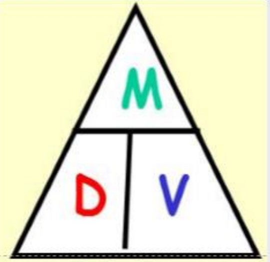
Force = Pressure x Area

Pressure = Force ÷ Area

Area = Force ÷Pressure



Mass = Density x Volume
Density= Mass ÷ Volume
Volume = Density ÷ Mass

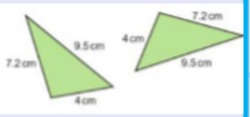


Unit 12 Higher Similarity and Congruence

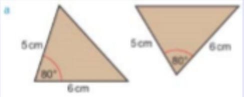
Congruent Triangles

Are exactly the same size and shape. Triangles are congruent when one of these conditions are true:

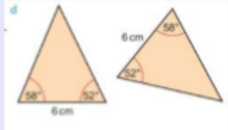
- SSS – all three sides are equal.



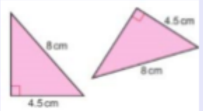
- SAS – two sides and included angle are equal.



- AAS – two angles and corresponding side are equal.



- RHS – right angle, hypotenuse and another side are equal.



V67

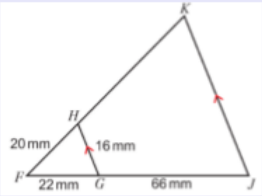
You need to prove it by using one of the above reasons.

Similarity

V291

Shapes are similar when one shape is an enlargement of each other. Corresponding sides are in the same ratio. Corresponding angles are equal. When comparing two similar shapes, a scale factor can be found. This scale factor helps to find missing sides of the shape.

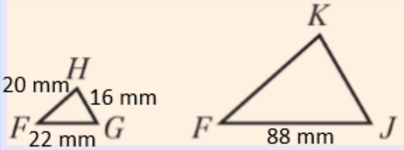
Similar Triangles



Prove FGH and FJK are similar.

Angle F occurs in both triangles. Therefore the same.

Draw the triangles separately.



FGH = FJK as corresponding angles.

FHG = FKJ as corresponding angles.

Therefore all angles are equal so triangle is similar.

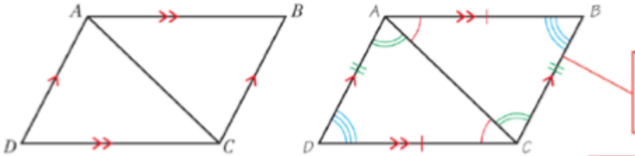
Congruence is used to solve problems and prove that shapes are the same.

To prove it: write a series of logical statements. Each statement needs must be supported by a mathematical reason.

Proving Geometric Congruence

V66

ABCD is a parallelogram. Prove triangle *ABC* is congruent to *ADC*.



Mark all equal angles and sides.

Length *AB* = length *CD* because opposite sides in a parallelogram are equal. State why $AB = CD$

Length *BC* = length *AD* because opposite sides in a parallelogram are equal. State why $BC = AD$

Length *AC* is common to both triangles.

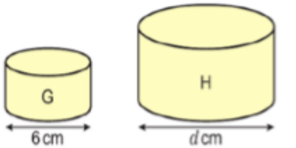
So triangle *ABC* is congruent to triangle *ADC* (SSS). State the condition used to prove congruence.

Similarity in 3D shapes

If a shape is enlarged by a linear scale factor of k, the area of the shape is enlarged by scale factor of k^2.

If a shape is enlarged by a linear scale factor of k, the volume of the shape is enlarged by scale factor of k^3.

Cylinders G and H are similar.
The diameter of G is 6 cm.
The volume of G is 108 cm³. The volume of H is 256 cm³
Work out the diameter *d* of cylinder H.



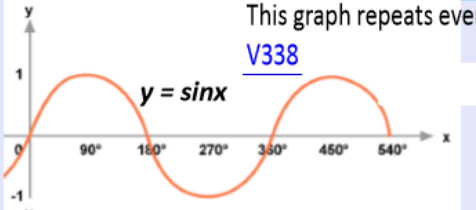
V293a
V293b

$$\text{Volume scale factor} = \frac{\text{large}}{\text{small}} = \frac{256}{108} = \frac{64}{27} = k^3$$

$$k = \sqrt[3]{\frac{64}{27}} = \frac{\sqrt[3]{64}}{\sqrt[3]{27}} = \frac{4}{3}$$

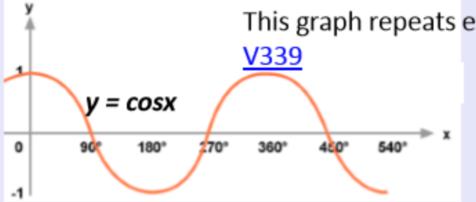
$d =$

Unit 13 Higher More Trigonometry



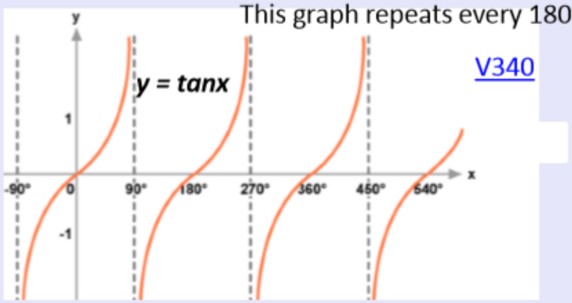
This graph repeats every 360⁰.

V338



This graph repeats every 360⁰.

V339



This graph repeats every 180⁰.

V340

Area of a triangle

$$\frac{1}{2}absinC$$

To be used when you can't use: $\frac{1}{2}$ base x height

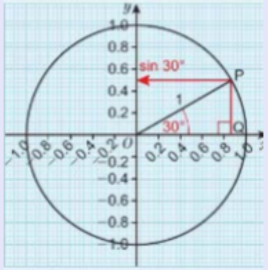
Transforming trigonometric graphs

y= f(x) is a function where x is the input. The output is *y* or *f(x).*

- *y= -f(x)* is a reflection in the *x*-axis.
- *y = f(-x)* is a reflection in the *y*-axis.
- *y=-f(-x)* is a reflection in the *y* and *x* axis. It is equivalent to a rotation of 180⁰ about the origin.
- *y = f(x + a)* is a translation by $\binom{-a}{0}$
- *y=af(x)* is a vertical stretch by scale factor *a*, parallel to the *y*-axis.
- *Y=f(ax)* is a horizontal stretch by the scale factor $\frac{1}{a}$

V323

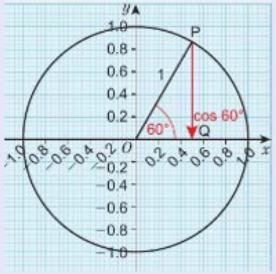
Sine function



The diagram shows a circle with radius 1 and centre (0,0). The length of PQ gives the sine of the angle.

$\sin30^0 = \frac{PQ}{1}$ = PQ = 0.5

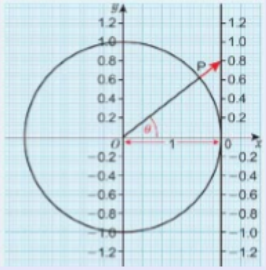
Cosine function



The diagram shows a circle with radius 1 and centre (0,0). The length of OQ gives the cosine of the angle.

$\cos60^0 = \frac{OQ}{1}$ = OQ = 0.5

Tangent function



The diagram shows a circle with radius 1 and centre (0,0). Extend OP to hit the tangent. This gives a value for tanθ.

$\tan\theta = \frac{0.8}{1}$ = 0.8

The sine rule

$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ To find a side.

V333

$\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$ To find an angle.

Can be used in any triangle. You need to know one angle and the opposite side. Then:

- If you know another angle, you can calculate its opposite side.
- If you know another side, you can calculate the opposite angle.

The cosine rule

V335 V336

$a^2 = b^2 + c^2 - 2bc$cosA To find a side.

cosA = $\frac{b^2 + c^2 - a^2}{2bc}$ To find an angle.

Can be used in any triangle. Use it to find:

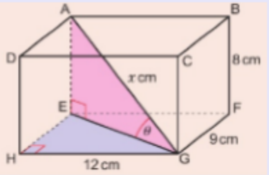
- The length of a side if you know two sides and the included angle
- An unknown angle if you know all three sides.

3D Pythagoras

V259

A plane is a flat surface.

EFGH is a horizontal plane. AEG is a vertical plane. AG is the diagonal named *x*. To calculate the value of *x*, you need to find the value of EG using Pythagoras' Theorem.



EG: $\sqrt{12^2 + 9^2}$ = 15cm

x: $\sqrt{15^2 + 8^2}$ = 17cm