Unit 11 Multiplicative Reasoning V236 V384 V385 V254 V255

In **compound interest** the interest earned each year is added to money in the account and earns interest the next year.
Most interest rates are compound interest rates.

You can calculate an amount after n years' compound interest using the formula

$$\text{amount} = \text{initial amount} \times \left(\frac{100 + \text{interest rate}}{100}\right)^n$$

If y is directly proportional to x, $y \propto x$ and $y = kx$, where k is a number, called the **constant of proportionality**.

Where k is the constant of proportionality:

- if y is proportional to the square of x then $y \propto x^2$ and $y = kx^2$
- if y is proportional to the cube of x then $y \propto x^3$ and $y = kx^3$
- if y is proportional to the square root of x then $y \propto \sqrt{x}$ and $y = k\sqrt{x}$

These are three kinematics formulae:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken

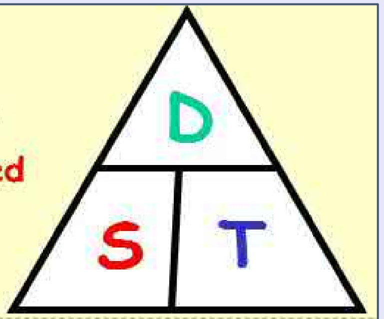
Multiplicative means involving multiplication or division

Key Words
Velocity
Acceleration
Force
Pressure

Distance = Time X Speed

Speed = Distance ÷ Time

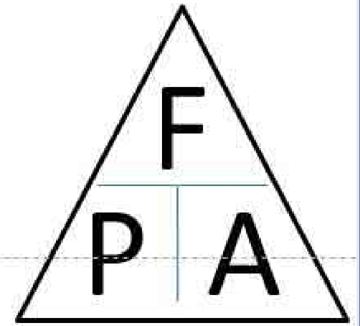
Time = Distance ÷ Speed



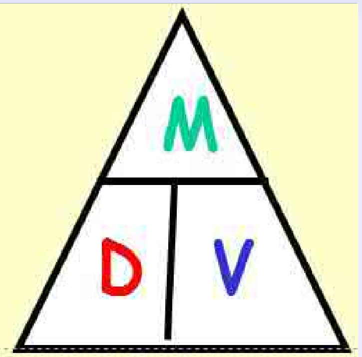
Force = Pressure x Area

Pressure = Force ÷ Area

Area = Force ÷Pressure



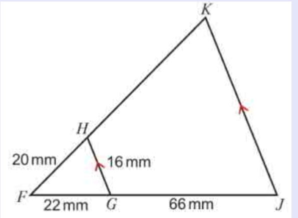
Mass = Density x Volume
Density= Mass ÷ Volume
Volume = Density ÷ Mass



Unit 12 Higher Similarity and Congruence

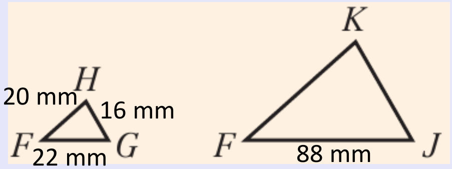
Similar Triangles

Prove FGH and FJK are similar.



Angle F occurs in both triangles. Therefore the same.

Draw the triangles separately.



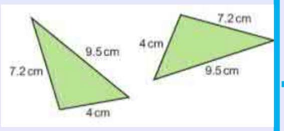
FGH = FJK as corresponding angles.

FHG = FKJ as corresponding angles.
Therefore all angles are equal so triangle is similar.

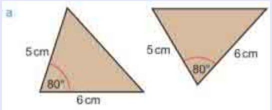
Congruent Triangles

Are exactly the same size and shape. Triangles are congruent when one of these conditions are true:

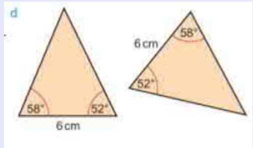
- SSS – all three sides are equal.



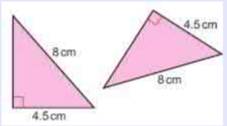
- SAS – two sides and included angle are equal.



- AAS – two angles and corresponding side are equal.



- RHS – right angle, hypotenuse and another side are equal.



V67

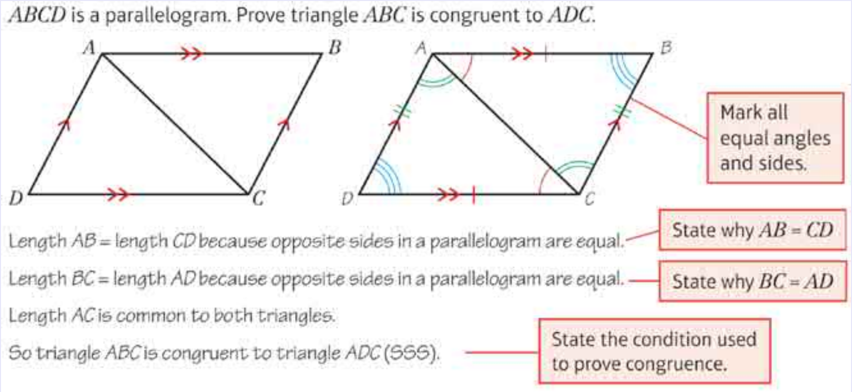
You need to prove it by using one of the above reasons.

Congruence is used to solve problems and prove that shapes are the same.

To prove it: write a series of logical statements. Each statement needs must be supported by a mathematical reason.

Proving Geometric Congruence

V66



Similarity

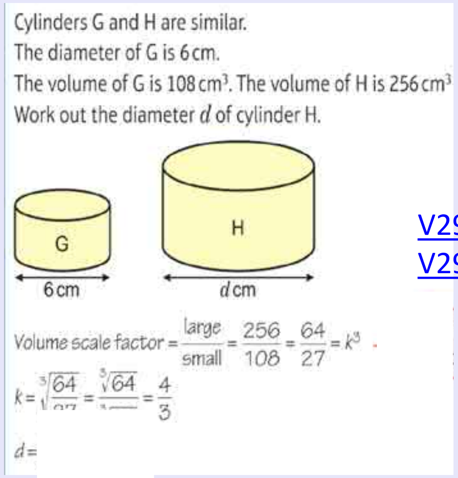
V291

Shapes are similar when one shape is an enlargement of each other. Corresponding sides are in the same ratio. Corresponding angles are equal. When comparing two similar shapes, a scale factor can be found. This scale factor helps to find missing sides of the shape.

Similarity in 3D shapes

If a shape is enlarged by a linear scale factor of k, the area of the shape is enlarged by scale factor of k^2.

If a shape is enlarged by a linear scale factor of k, the volume of the shape is enlarged by scale factor of k^3.



V293a
V293b

Unit 13 Higher More Trigonometry

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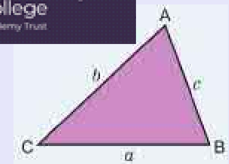
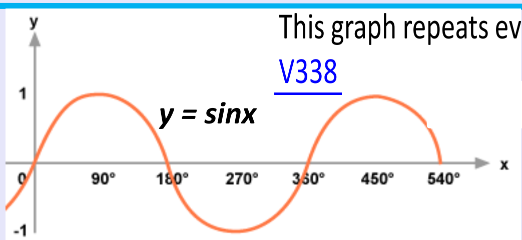


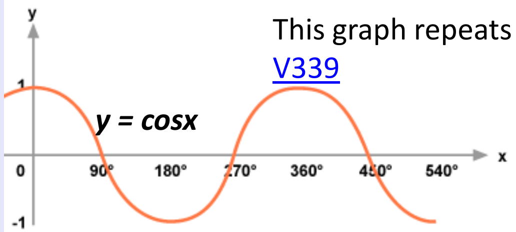
Transforming trigonometric graphs

y= f(x) is a function where x is the input. The output is *y* or *f(x)*.

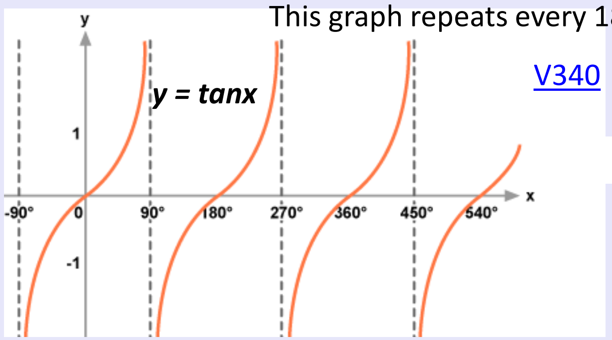
- *y= -f(x)* is a reflection in the *x*-axis.
- *y = f(-x)* is a reflection in the *y*-axis.
- *y=-f(-x)* is a reflection in the *y* and *x* axis. It is equivalent to a rotation of 180^0 about the origin.
- *y = f(x + a)* is a translation by $\binom{-a}{0}$ V323
- *y=af(x)* is a vertical stretch by scale factor *a*, parallel to the *y*-axis.
- *Y=f(ax)* is a horizontal stretch by the scale factor $\frac{1}{a}$



This graph repeats every 360^0.
V338



This graph repeats every 360^0.
V339



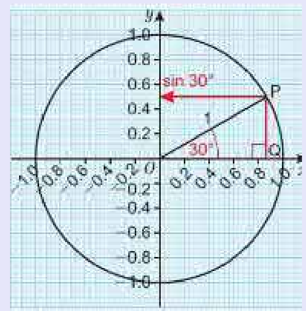
This graph repeats every 180^0.
V340

Area of a triangle

$$\frac{1}{2}absinC$$

To be used when you can't use: $\frac{1}{2}$ base x height

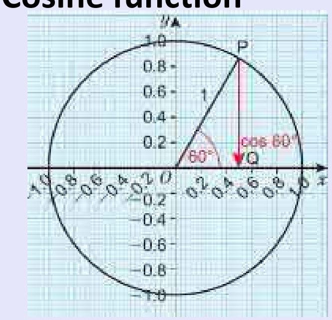
Sine function



The diagram shows a circle with radius 1 and centre (0,0). The length of PQ gives the sine of the angle.

sin30^0 = $\frac{PQ}{1}$ = PQ = 0.5

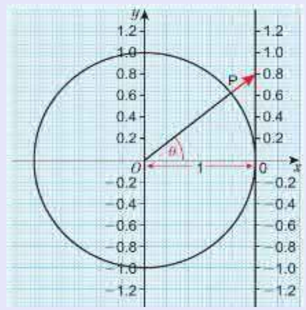
Cosine function



The diagram shows a circle with radius 1 and centre (0,0). The length of OQ gives the cosine of the angle.

cos60^0 = $\frac{OQ}{1}$ = OQ = 0.5

Tangent function



The diagram shows a circle with radius 1 and centre (0,0). Extend OP to hit the tangent. This gives a value for tanθ.

tanθ = $\frac{0.8}{1}$ = 0.8

The sine rule

$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ To find a side.

V333

$\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$ To find an angle.

Can be used in any triangle. You need to know one angle and the opposite side. Then:

- If you know another angle, you can calculate its opposite side.
- If you know another side, you can calculate the opposite angle.

The cosine rule

V335 V336

$a^2 = b^2 + c^2 - 2bccosA$ To find a side.

cosA = $\frac{b^2+c^2-a^2}{2bc}$ To find an angle.

Can be used in any triangle. Use it to find:

- The length of a side if you know two sides and the included angle
- An unknown angle if you know all three sides.

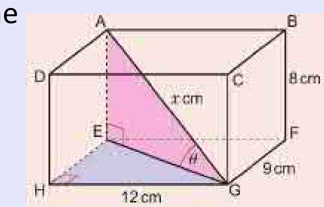
3D Pythagoras

V259

A plane is a flat surface.
EFGH is a horizontal plane. AEG is a vertical plane.
AG is the diagonal named x. To calculate the value of x, you need to find the value of EG using Pythagoras' Theorem.

EG: $\sqrt{12^2 + 9^2}$ = 15cm

x: $\sqrt{15^2 + 8^2}$ = 17cm



Unit 14 Higher Further Statistics V149 V159 V154 V281

Key Words

Population
Census
Sample
Bias
Random Sample
Strata

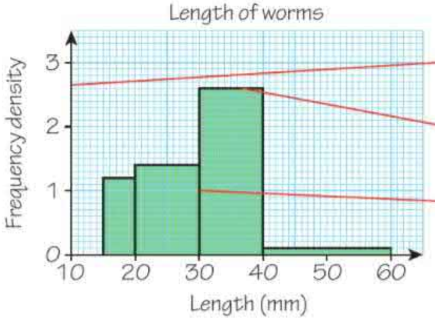
Histogram

The lengths of 48 worms are recorded in this table.

Length, x (mm)	$15 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 60$
Frequency	6	14	26	2

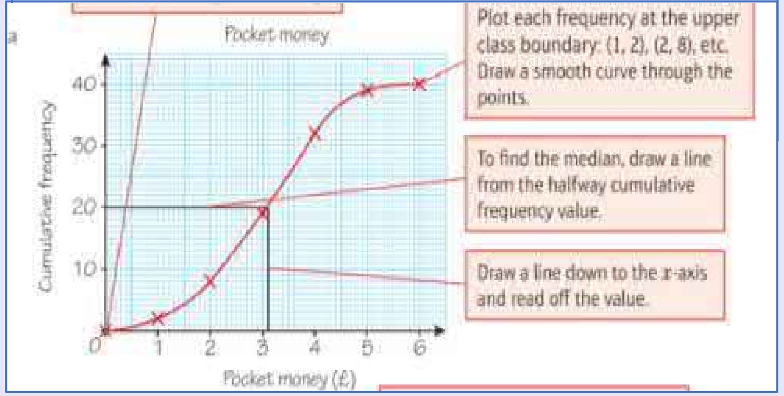
Draw a histogram to display this data.

$6 \div 5 = 1.2$, $14 \div 10 = 1.4$, $26 \div 10 = 2.6$, $2 \div 20 = 0.1$ — Work out the frequency density for each class



Cumulative Frequency

Pocket money, x (£)	Cumulative frequency
$0 < x \leq 1$	2
$0 < x \leq 2$	8
$0 < x \leq 3$	19
$0 < x \leq 4$	32
$0 < x \leq 5$	39
$0 < x \leq 6$	40



Stratified Sample

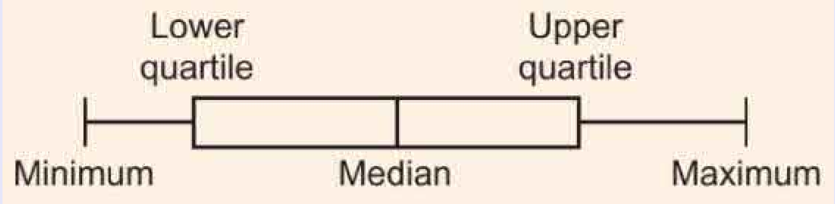
$$\frac{Sample}{Population} \text{ X Stratum Size}$$

Catch and Release

$$\frac{n}{N} = \frac{m}{M}$$

$$\text{So } N = \frac{n}{m} \text{ x } M$$

Box Plot

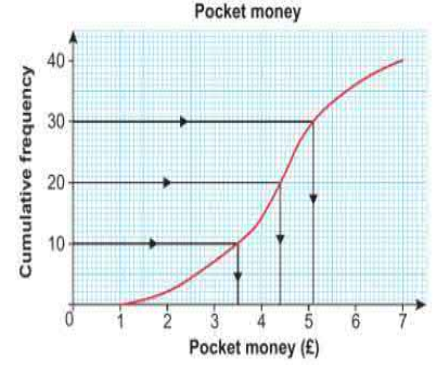


Range is Max – Min

Always Compare a measure of 'spread' and a value

The **median** and **quartiles** can be estimated from the cumulative frequency diagram. For a set of n data values

- the estimate for the **median** is the $\frac{n}{2}$th value
- the estimate for the **lower quartile** (LQ) is the $\frac{n}{4}$th value
- the estimate for the **upper quartile** (UQ) is the $\frac{3n}{4}$th value
- the **interquartile range** (IQR) = UQ – LQ

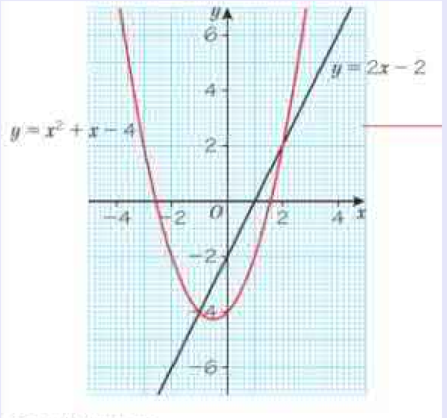


Unit 15 Higher Equations and Graphs

Quadratic Graphs

V180 V181 V276c VCubic

Solving Simultaneous Equations

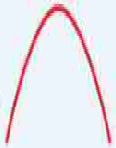


The solutions are
$x = 2, y = 2$ and $x = -1, y = -4$

The lowest or highest point of the parabola, where the graph turns, is called the **turning point**.

The turning point is either a minimum or maximum point.

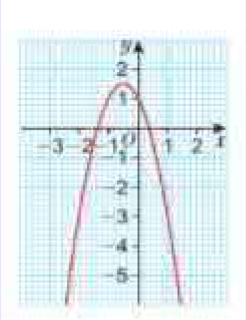
The x-values where the graph intersects the x-axis are the solutions, or **roots**, of the equation $y = 0$.

 is a minimum,  is a maximum

When a quadratic is written in completed square form $y = a(x + b)^2 + c$ the coordinate of the turning point is $(-b, c)$

To sketch a quadratic function

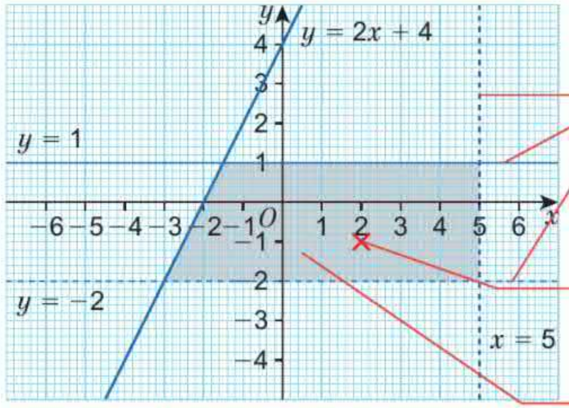
- Calculate the solutions to the equation '$y = 0$' (points of intersection with the x-axis).
- Calculate the point at which the graph crosses the y-axis.
- Find the coordinates of the turning point and whether it is a maximum or a minimum.



The quadratic equation $ax^2 + bx + c = 0$ is said to have no real roots if its graph does not cross the x-axis. If its graph just touches the x-axis, the equation has one repeated root.

Inequality Graphs

On a coordinate grid, shade the region that satisfies the inequalities
$x < 5$, $y \leqslant 2x + 4$, $y \leqslant 1$ and $y > -2$



Draw dotted lines $x = 5$ and $y = -2$
Draw solid lines $y = 2x + 4$, $y = 1$

Test a point. For $(2, -1)$
$y \leqslant 1$ and $y > -2$: the y-coordinate is -1
$x < 5$: the x-coordinate is 2
$2x + 4 = 8$: y-coordinate $\leqslant 8$

Shade the required region.

Cubic Graphs

A **cubic** function is one whose highest power of x is x^3.
It is written in the form $y = ax^3 + bx^2 + cx + d$

When $a > 0$ the function looks like

 or 

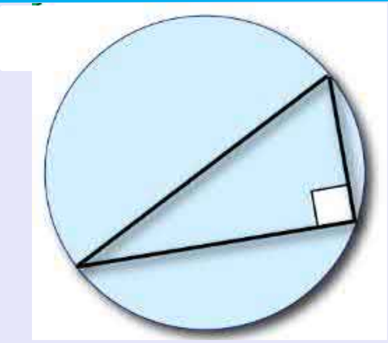
When $a < 0$ the function looks like

 or 

The graph intersects the y-axis at the point $y = d$
The graph's roots can be found by finding the values of x for which $y = 0$.

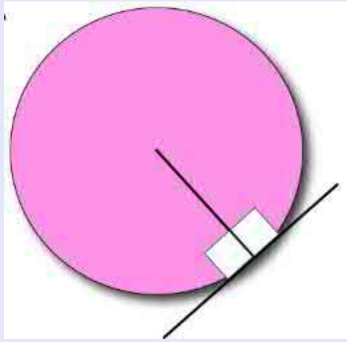
Unit 16 Higher Circle Theorems

V64 V65



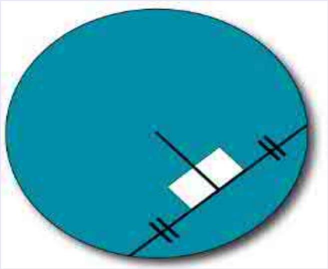
The angle in a semicircle is a right angle.

V65a



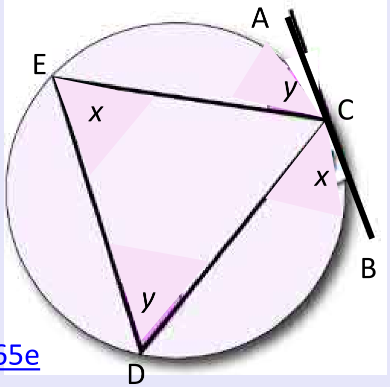
The angle between a **tangent** and **radius** is a right angle.

V65f



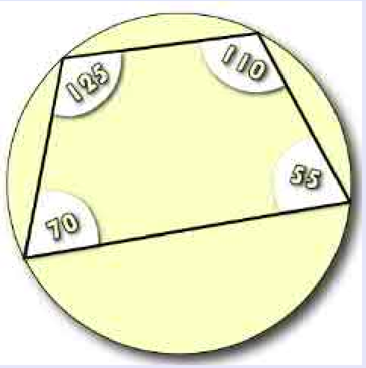
A **chord** is a straight line connecting two points on a circle.

The **perpendicular** from the centre of the circle to a chord **bisects** the chord and the line drawn from the centre of the circle to the **midpoint** of a chord is at right angles to the chord.



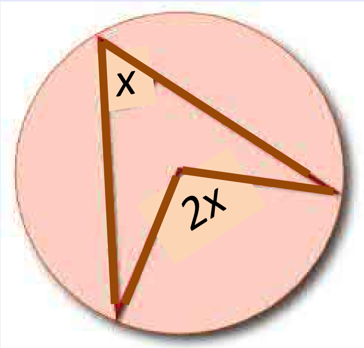
V65e

AB is a tangent to the circle. CD, DE and CE are **chords**. Angle ECA is the angle between the **tangent** and the chord in one segment. The other **segment** has angle CDE. This is the **alternate segment.** The angle between the chord and tangent is equal to the angle in the alternate segment.



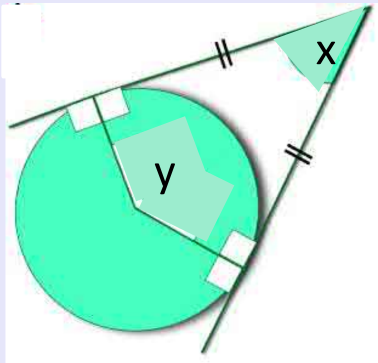
A cyclic quadrilateral with all four vertices on the circumference of the circle. Opposite angles add up to 180º.

V65d



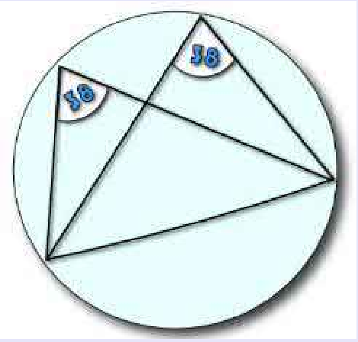
The angle at the centre of a circle is twice the angle at the circumference when both are subtended by the same arc.

V65b



Tangents drawn from a point outside the circle are equal in length.

$x + y = 180$



Angles in the same segment and standing on the same chord are always equal.

V65c

Unit 17 Higher More Algebra V8 V21 V22 V23 V24 V365 V370

You can change the subject of a formula by isolating the terms involving the new subject.

When the letter to be made the subject appears twice in the formula you will need to factorise.

You may need to factorise before simplifying an algebraic fraction:

- Factorise the numerator and denominator.
- Divide the numerator and denominator by any common factors.

You may need to factorise the numerator and/or denominator before you multiply or divide algebraic fractions.

To add or subtract algebraic fractions, write each fraction as an equivalent fraction with a common denominator.

To find the lowest common denominator of algebraic fractions, you may need to factorise the denominators first.

To rationalise the fraction $\frac{1}{a \pm \sqrt{b}}$, multiply by $\frac{a \mp \sqrt{b}}{a \mp \sqrt{b}}$.

To show a statement is an identity, expand and simplify the expressions on one or both sides of the equals sign, until the two expressions are the same.

A function is a rule for working out values of y when given values of x e.g. $y = 3x$ and $y = x^2$

The notation $f(x)$ is read as 'f of x'.

fg is the composition of the function f with the function g. To work out $fg(x)$, first work out $g(x)$ and then substitute your answer into $f(x)$.

The inverse function reverses the effect of the original function. $f^{-1}(x)$ is the inverse of $f(x)$.

To prove a statement is not true you can find a **counter-example** – an example that does not fit the statement.

For an algebraic proof let *n* represent any integer

Even number	$2n$
Odd number	$2n + 1$ or $2n - 1$
Consecutive numbers	$n, n + 1, n + 2, \ldots$
Consecutive even numbers	$2n, 2n + 2, 2n + 4, \ldots$
Consecutive odd numbers	$2n + 1, 2n + 3, 2n + 5, \ldots$

Unit 18 Higher Vectors and Proof V353 V353a

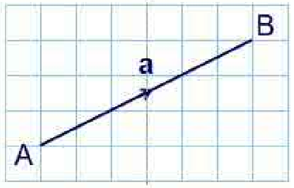
A **vector** is a quantity that has both size (or magnitude) and direction.

Examples of vector quantities are:
- displacement
- velocity
- force

A **scalar** is a quantity that has size (or magnitude) only.

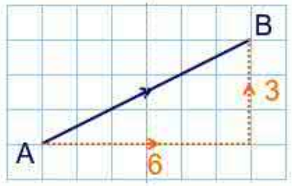
Examples of scalar quantities are:
- length
- speed

Vectors are written as **bold** lower case letters: **a**, **b**, **c**. When handwriting, underline the letter: a̲, b̲, c̲ .



This vector goes from the point A to the point B.

We can write this vector as $\overrightarrow{AB}$.



To go from the point A to the point B we must move 6 units to the right and 3 units up.

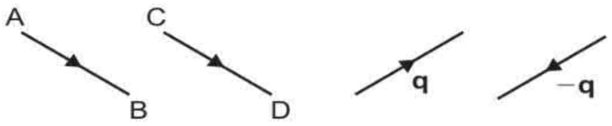
We can represent this movement using a **column vector**.

$\overrightarrow{AB} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$

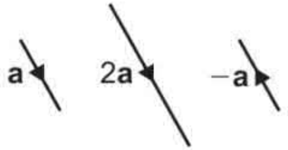
This is the horizontal component. It tells us the number of units in the x-direction.

This is the vertical component. It tells us the number of units in the y-direction.

If $\overrightarrow{AB} = \overrightarrow{CD}$ then the line segments AB and CD are equal in length and are parallel. $\overrightarrow{AB} = -\overrightarrow{BA}$



2**a** is twice as long as **a** and in the same direction.
−**a** is the same length as **a** but in the opposite direction.



With the origin O, the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are called the **position vectors** of the points A and B. In general, a point with coordinates (p, q) has position vector $\begin{pmatrix} p \\ q \end{pmatrix}$.

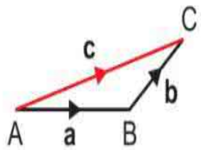
In general, if the vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is multiplied by the scalar k, then

$$k \times \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$$

For example, $3 \times \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} -6 \\ 15 \end{pmatrix}$

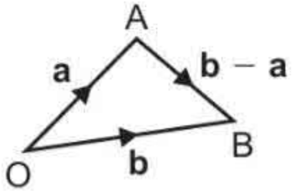
When a vector is multiplied by a scalar the resulting vector is either parallel to the original vector or lies on the same line.

Triangle law for vector addition: Let $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$ and $\overrightarrow{AC} = \mathbf{c}$. Then $\mathbf{a} + \mathbf{b} = \mathbf{c}$ forms a triangle.

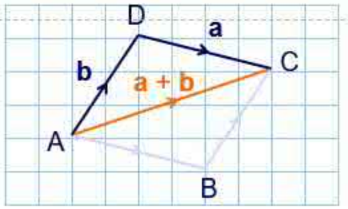


When $\mathbf{c} = \mathbf{a} + \mathbf{b}$ the vector **c** is called the **resultant vector** of the two vectors **a** and **b**.

When $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \mathbf{b} - \mathbf{a}$.



Suppose $\mathbf{a} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$



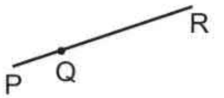
From this diagram we can see that

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \mathbf{a} + \mathbf{b}$

Also

$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} = \mathbf{b} + \mathbf{a}$

$\overrightarrow{PQ} = k\overrightarrow{QR}$ shows that the lines PQ and QR are parallel. Also they both pass through point Q so PQ and QR are part of the same straight line. P, Q and R are said to be **collinear** (they all lie on the same straight line).



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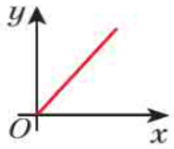
Unit 19 Higher Proportion and Graphs V345 V255 V254

When a graph of two quantities is a straight line through the origin, one quantity is directly proportional to the other.



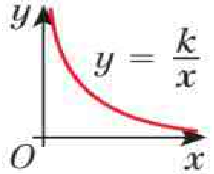
The symbol $\propto$ means 'is directly proportional to'.

If y is directly proportional to x, $y \propto x$ and $y = kx$, where k is a number, called the **constant of proportionality**.

Where k is the constant of proportionality:

- if y is proportional to the square of x then $y \propto x^2$ and $y = kx^2$
- if y is proportional to the cube of x then $y \propto x^3$ and $y = kx^3$
- if y is proportional to the square root of x then $y \propto \sqrt{x}$ and $y = k\sqrt{x}$

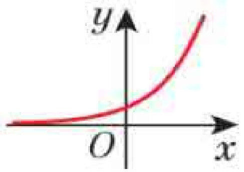
When y is **inversely proportional** to x, $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$



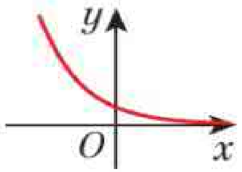
The tangent to a curved graph is a straight line that touches the graph at a point. The gradient at a point on a curve is the gradient of the tangent at that point.

Expressions of the form a^x or a^{-x}, where $a > 1$, are called **exponential functions**.

The graph of an exponential function has one of these shapes.



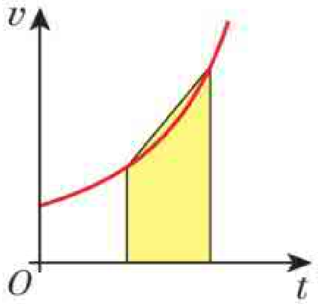
$y = a^x$ where $a > 1$ or
$y = b^{-x}$ where $0 < b < 1$
exponential growth



$y = a^{-x}$ where $a > 1$ or
$y = b^x$ where $0 < b < 1$
exponential decay

Exponential graphs intersect the y-axis at (0, 1) because $a^0 = 1$ for all values of a.

The area under a velocity–time graph shows the displacement, or distance from the starting point. To estimate the area under a part of a curved graph, draw a chord between the two points you are interested in, and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an estimate for the area under this part of the graph.



The gradient of the chord gives the average rate of change

Higher: Transformation of Graphs Corbett Maths link:[Transformations-of-graphs]

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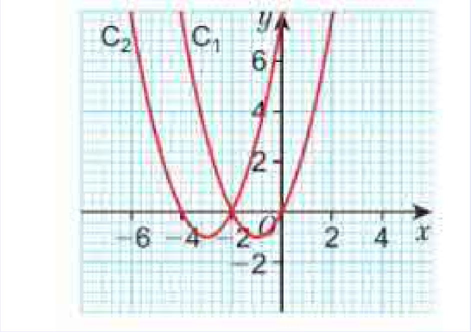
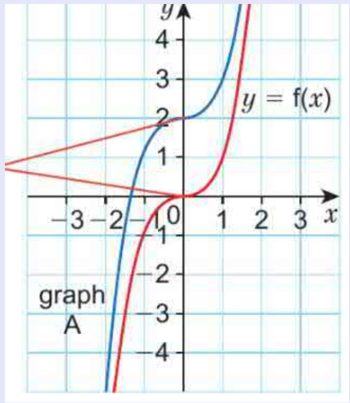
The graph of $y = f(x)$ is transformed into the graph of:
$y = f(x) + a$ by a translation of a units parallel to the y-axis or a translation by $\begin{pmatrix} 0 \\ a \end{pmatrix}$

The graph of $y = f(x)$ is transformed into the graph of:
$y = f(x) + a$ by a translation of a units parallel to the y-axis or a translation by $\begin{pmatrix} 0 \\ a \end{pmatrix}$

$y = f(x + a)$ by a translation of $-a$ units parallel to the x-axis or a translation by $\begin{pmatrix} -a \\ 0 \end{pmatrix}$

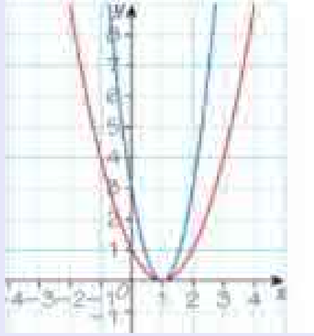




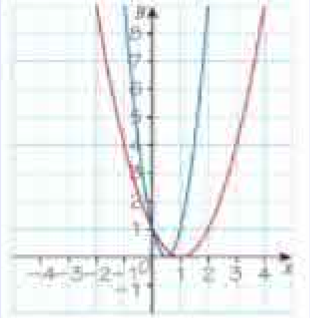
$y = f(-x)$ by a reflection in the y-axis

$y = -f(x)$ by a reflection in the x-axis

$y = af(x)$ by a stretch of scale factor a parallel to the y-axis



$y = f(ax)$ by a stretch of scale factor $\frac{1}{a}$ parallel to the x-axis



edexcel

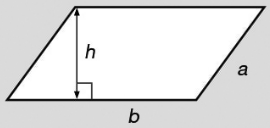
Edexcel GCSE (9-1) Maths: need-to-know formulae

www.edexcel.com/gcsemathsformulae

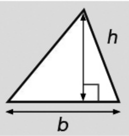
Areas

Rectangle = $l \times w$

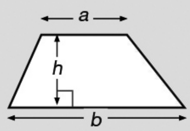
Parallelogram = $b \times h$



Triangle = $\frac{1}{2} b \times h$



Trapezium = $\frac{1}{2}(a + b)h$

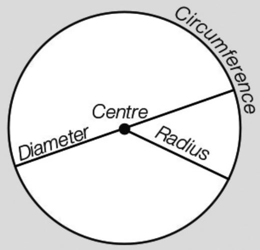


Circles

Circumference = $\pi \times$ diameter, $C = \pi d$

Circumference = $2 \times \pi \times$ radius, $C = 2\pi r$

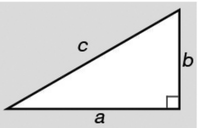
Area of a circle = π x radius squared, $A = \pi r^2$



Pythagoras

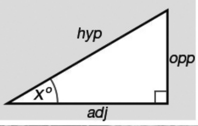
Pythagoras' Theorem

For a right-angled triangle,
$a^2 + b^2 = c^2$



Trigonometric ratios (*new to F*)

$\sin x^\circ = \frac{\text{opp}}{\text{hyp}}$, $\cos x^\circ = \frac{\text{adj}}{\text{hyp}}$, $\tan x^\circ = \frac{\text{opp}}{\text{adj}}$



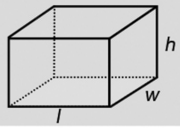
Quadratic equations

The Quadratic Equation

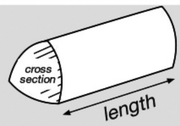
The solutions of $ax^2 + bx + c = 0$,
where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Volumes

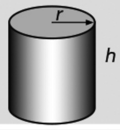
Cuboid = $l \times w \times h$



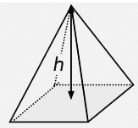
Prism = area of cross section $\times$ length



Cylinder = $\pi r^2 h$



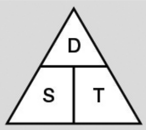
Pyramid = $\frac{1}{3} \times$ area of base $\times h$



Compound measures

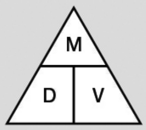
Speed

speed = $\frac{\text{distance}}{\text{time}}$



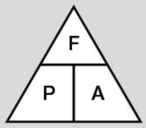
Density

density = $\frac{\text{mass}}{\text{volume}}$



Pressure

pressure = $\frac{\text{force}}{\text{area}}$

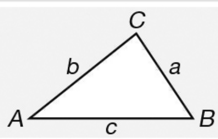


Trigonometric formulae

Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$



Foundation tier formulae

Higher tier formulae